Analysis of flow instability in two-phase natural circulation loop using drift-flux model

by

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NOVEMBER, 2002

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A thesis submitted
in partial fulfillment of the requirement
for the degree of

Master of Technology

by

Prashant Dubey

to the

Department of Nuclear Engineering and Technology

Indian Institute of Technology, Kanpur

November,2002

CERTIFICATE

It is certified that the work contained in this thesis entitled **"Analysis of flow instability in two-phase natural circulation loop using drift-flux model"** by **Prashant Dubey** has been carried out under our supervision and this work has not been submitted elsewhere for a degree.

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Acknowledgement

It was immense pride to be a student of the Nuclear Engg. and Technology Programme associating with the dedicated teaching community, which I will treasure for the rest of my life. I express my sincere gratitude and regards to my supervisor Dr. Ashok Khanna for their inspiring guidance, and invaluable suggestions in each and every step of this work.I express my regards, thanks, to my supervisor Dr. P.K.Vijayan for his valuable suggetion and encouragement that I received during my thesis work at BARC. I express my deep sense of gratitude and thanks to Dr. A.K.Nayak for his excellent guidance, invaluable suggestion and co-operation

I express my sincere thanks to my friends Kamal, Mrinal, Anuarag, Gaurav and Manas for their laudable help. They provided me the great support and lovely family atmosphere at this place.

The love and constant encouragement from my whole family is an invaluable asset that I got through out the period of my study. Without their encouragement nothing in my life would have been possible.

I.I.T. Kanpur

Prashant Dubey

November, 2002

Abstract

In two-phase natural circulation loop, the circulating fluid removes the heat from the heat source. With natural circulation being the designed heat removal mode, the flow may experience various types of thermohydraulic instabilities, such as flow region transition, boiling crisis, Ledinegg instability, density wave oscillations, Pressure drop oscillations, acoustic oscillations etc. However most common among these instabilities is density wave oscillation. So present work has analyzed this instability.

For this purpose computer code TINFLOS-D (based on drift flux mode) has been obtained by modifying the computer code TINFLOS (based on homogeneous model). The code TINFLOS-D solves the four conservations equations using Linear-stability analytical technique. The result obtained from the code has been validated for stability with four single channel two-phase natural circulation loop. Two loops HPNCL and Apsara are in BARC. Code has also been validated with two other natural-circulation loops reported in literature (Furuteara's and Bergles). HPNCL has been validated for steady state also. Parametric sensitivity analysis has been done for low quality HPNCL for lower threshold of instability. It has been found that natural circulation flow rate first increases then decreases with increase in system pressure. The instability reduces with increases in system pressure, power and drift velocity. The instability also reduces with increase in core inlet resistance and core outlet resistance. The instability increases with increase in riser height, downcomer diameter and inlet subcooling.

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Nomenclature

A	Cross sectional area (m^2)
C_k	kinematic wave velocity (m/s)
D	hydraulic diameter (m)
f	darcy friction factor
g	acceleration due to gravity (m/s^2)
h	enthalpy (J/kg)
h_{fg}	latent heat of vaporization (J/kg)
j	volumetric flux density of mixture (m/s)
j_f	volumetric flux density of liquid (m/s)
j_g	volumetric flux density of vapour (m/s)
K_l	loss coefficient
L	length of the section (m)
p	pressure (bar)
P_h	heated perimeter(m)
q''_w	heat flux (W/m^2)
s	stability parameter
t	time (s)
T	temperature (K)
v	specific volume (m^3/kg)
V_{fi}	velocity of single phase liquid (m/s)
V_{gj}	drift velocity of vapour (m/s)
V_m	mass velocity of centre of mass of the mixture (m/s)
w	mass flow rate (kg/s)
z	axial distance (m)

Greek Symbols

α	void fraction
τ	residence time of fluid in single phase region (s)
Γ_g	vapour generation rate (kg/m^3-s)
λ	non boiling length in core (m)
Δ	difference
ρ	density (kg/m^3)
Φ	phase

Subscripts

c	core
f	liquid
g	vapour
m	mixture
r	relative
s	single phase
ss	steady state
sub	subcooling

CHAPTER 1

LITERATURE REVIEW

1.1 Introduction

In a natural circulation loop the circulating fluid removes the heat from a heat source and transports it to heat sink, the fluid circulation being established due to the buoyancy force. The buoyancy force is the driving force. Buoyancy force is result of thermally induced density differences in a gravitational field. The heat sink is located at the higher elevation than the heat source. Such loops find application in many engineering fields such as, gas turbine blade cooling, solar water heater, geothermal systems, electrical machine rotor cooling, transformer cooling, nuclear reactor core cooling etc.

Present generation boiling water reactors utilize forced circulation loops for heat transfer from core to the steam generator. But this method has the following disadvantages:

1. Usage of pump is costly
2. If pump break down occurs then dissipation of fission heat is hampered. This results in tremendous accumulation of heat and may cause core melt down in the reactor, and
3. Back up safety measures have to be used.

Hence next generation of boiling water reactors should have natural circulation of primary coolant under normal operational situations. Fig. 1.1 shows the schematic diagram of a natural circulation loop. The loop essentially consists of a tubular heated section, a riser, a steam drum or a separator, a condenser, a downcomer, and upper- lower horizontal sections. The steam collected in separator or steam drum goes to condenser, where it is condensed and then sent back to the separator. This loop differs from forced circulation loop only by the absence of pump.

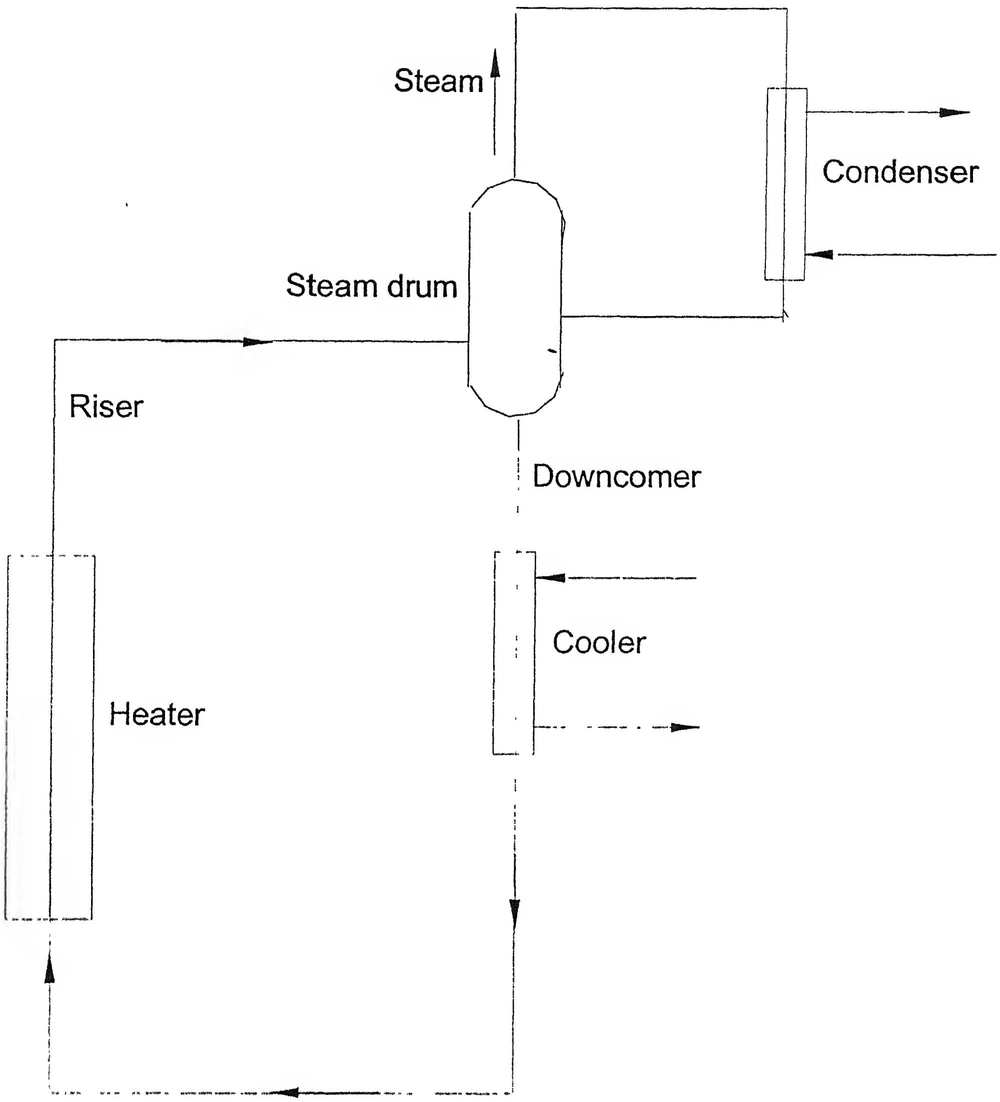


Fig 1.1 Schematic of Natural Circulation Loop

In a two-phase natural circulation loop since heat is removed by natural circulation, it is inherently safe. This method of cooling enhances passive safety.

Many proposed advanced designs of nuclear power reactors use two-phase natural circulation of primary coolants. However a major problem with these loops is occurrence of flow instabilities. Flow instabilities are undesirable in boiling, condensing and other two-phase flow systems for several reasons.

Boure et. al. [1] divided these instabilities into two classes static instability and dynamic instability.

Static instability is one where a small change in one of the independent variables leads to a large change in one of the dependent variables. Examples of static instability are listed below:

(a) Flow regime transition: Cyclic flow pattern transitions and flow pattern variation occurs.

(b) Boiling crisis: Bubbly flow has less void but higher Δp than that of annular flow characterized by the Ineffective removal of heat from the heated surface leads to wall temperature excursion and flow oscillation .

(c) The Ledinegg instability or flow excursion instability : Flow undergoes sudden, large amplitude excursion to a new, stable operating condition, due to the internal pressure gradient being less than the external pressure gradient (imposed by the pump).

(d) Geysering: an erratic boiling region exists, where the the surface temperature moves between boiling and natural circulation in irregular cycles, due to the presence of gas; effect disappears at higher heat fluxes and higher pressures

Dynamic instability: a flow is subjected to dynamic instability when inertia and other feedback effects have an essential part in the process.

(a) Density wave oscillations: Occurs due to delay and feedback effects between flow rate, density and Δp.

(b) Pressure drop oscillations: These are very low frequency (0.1 Hz) oscillations; These oscillations occurs due to dynamic interaction between channel and the compressible volume, which is initiated by flow excursions.

(c) Acoustic oscillations: These are high frequency (10-100 Hz) oscillations related to time required for pressure wave propagation; occurs due to resonance of pressure waves.

(d) Thermal oscillations: These oscillations occur in film boiling due to interaction of variable heat transfer coefficients with flow dynamics.

(e) BWR instability: It is strong only for small fuel time constant and under low pressures; occurs due to interaction of void reactivity coupling with the flow dynamics and heat transfer.

(f) Parallel channel instability: there are various mode of flow redistribution; which occurs due to interaction between small number of parallel channels.

However most common among these instabilities is density wave oscillations. Present work analyses this instability. If a flow perturbation w' is introduced at the inlet to the boiler tube, there will be perturbation in the single-phase pressure drop $\Delta p'_{1-\Phi}$ in the subcooled region, which is in phase with the flow perturbation. There will however be a lag in the movement of the boiling boundary, which in turn leads to an increasing lag in perturbation in the density and velocity of two-phase region. This leads to a lag in two-phase pressure drop perturbation $\Delta p'_{2-\Phi}$. If the lag in two-phase pressure drop perturbation $\Delta p'_{2-\Phi}$ at a particular frequency is 180°, the two-perturbations ($\Delta p'_{1-\Phi}$ and $\Delta p'_{2-\Phi}$) cancel out leaving same constant pressure drop with an oscillatory flow. This is the threshold of instability. When the perturbation crosses this threshold then, the two-phase pressure drop, at this particular frequency

lags by180°, and has amplitude greater than that in single-phase region, the overall restriction of constant pressure imposed by the headers will lead to divergent flow oscillations.

Rizwan-Uddin [2] identified that the axial variation in the mixture velocity is the origin of density wave instability in two-phase flow.

It can be now said that the mechanism behind this type of oscillations is now well understood in addition to many other type of instabilities, which are cited earlier. However experimental and analytical Investigations are still being carried out (a) to predict the onset of instability because threshold of instability and stability behavior is found to vary by differences in boiling loop geometry and their operating conditions, and (b) to understand the characteristics of oscillations under unstable operations.

1.2 Experimental investigations

1.2.1 In natural circulation loops

Experimental investigations in two-phase natural circulation loops having single heated boiling channel have been carried out by several workers[3-7].They observed density wave instability, which was found to increase with increase in channel exit restriction, inlet subcooling and decrease in system pressure, channel inlet restriction and down comer level.

Bergles et. al. [3] has carried out experimental investigations in a natural circulation loop similar to thermosyphon reboiler used for chemical processing and waste heat recovery. They also observed that when the liquid level in downcomer is above the riser exit, the system is stabilised. Lee and Ishii [4] found that the non-equilibrium between the two phases such as during flashing, created flow instability in the loop. Kyung and Lee [5] investigated the flow characteristics in an open two-phase natural circulation loop using freon as the test fluid. They observed three different modes of oscillation with increase in heat flux:

(a) Periodic flow oscillations characterized by an incubation period of boiling

(b) Continuous circulation, which is maintained with churn/wispy-annular flow pattern. This was found to be a stable operation mode in which the flow increases with heat flux and then decreases with further increase in heat flux.

(c) Periodic flow oscillations characterised by continuous boiling inside the heater section. (i.e. there is no incubation period); and void fluctuates between 0.6 to 1.0 repeatedly. In this mode mean circulation rate was found to decrease with increase in heat flux although mean void kept on increasing.

Jiang et.al. [6] Observed three different kinds of instabilities-geysering, flashing and density wave oscillations during the start-up of natural circulation loop. Wu.et.al. [7] observed that flow oscillatory behavior was dependent on heating power and inlet subcooling. Depending on the operating conditions these oscillations can be periodic or chaotic. Mathisen [8] has carried out experimental investigation in natural circulation loop having parallel heated boiling channel. He observed that increase of input power reduces the flow stability. He also found that a possible remedy is an increase of system pressure to stabilize the flow at high input power. Fukuda and Kobri [9] have classified density wave instability as type I and type II depending on the operating condition of the system.Type I instability occurs under low quality condition due to the dominance of gravitational pressure loss term. Type II instability occurs under high quality condition due to the dominance of frictional pressure loss term. Aritomi et.al. [10] observed three kinds of instabilities during the power raising process of a natural circulation loop with twin boiling channels, such as geysering, in-phase natural circulation oscillation and out-of-phase density wave oscillations. Saha et. al. [11] observed steady state and stability behavior of a natural circulation loop. They observed that geysering like instability was occurring during start up which developed into type I instability with increase in power. They also observed that system stability increases with increase in inlet loss coefficients and with decrease in inlet subcooling.

1.2.2 In boiling forced circulation loops

Similar experiments were carried out to study the flow instability in forced circulation loop. Aritomi et. al. [12] found that the flow oscillations in parallel channels were independent of magnitude and nature of disturbance at the inlet . They also found that when flow conditions differed between the channels (but where the individual characteristic frequencies and inertia masses were approximately equal) the instability behavior almost agreed with that of under the average operating conditions. The system was found to be more stable with increase in difference of flow conditions between the channels. Takitani and Takemura [13] observed that the phase difference between inlet and outlet flow rates in the boiling channel was 180 degree. They also observed that threshold of instability was unaffected by presence or absence of superheated steam. Nakanishi et. al. [14] observed both fundamental and higher mode of oscillations in their experiment. Wang et. al. [15] observed density wave oscillations up to 100 bar in their experiments. Xiao et. al. [16] observed pressure drop and thermal oscillations in addition to the density wave oscillations which occurred both at low and high flow quality in their experiment. Their experiment also revealed that density wave oscillations can appear up to 192 bar and disappear above 207 bar. Fukuda et. al. [17] conducted experiments in multiple parallel channels (upto 7 channels) with short or long riser pipes. They observed type 1 density wave instability in both the cases. Collins and Gacesa [18] tested the effect of mass velocity and power on flow oscillation frequency in a 19 rod bundle with steam-water flow at 800 psia. They found that oscillation frequency increases with increase in mass velocity as well as increase in power input to the channel. Dijkman[19] had tested the effect of cosine heat flux distribution. He found that cosine heat flux distribution stabilizes the flow.

1.3 Theoretical investigations in boiling systems

Several studies based on linear and non-linear analysis have been carried out in the past to investigate the flow instability in the two-phase flow systems. These analyses solve the energy and continuity equations to obtain the

enthalpy distribution in the heated region. The momentum equation is then integrated around the loop with this enthalpy distribution. In the linear analytical technique the governing equations are linearised by perturbing equations around the steady state and solved analytically to obtain the characteristic equation and the stability of system is investigated from the root of the characteristic equation. In the non-linear analysis, the governing equations are solved numerically by using finite difference method .The linear analysis takes less CPU time to evaluate the threshold of instability. Hence, it is useful when analysing the mechanism of instability by plotting stability maps. But the accuracy of this method is limited. Beyond the threshold, non-linear oscillations may appear which can not be predicted by the linear stability technique. However, the problem with non-linear analysis is the numerical diffusion which may appear in the calculations and then it is difficult to interpret the physical instability of the system.

1.3.1 Linear analysis

Investigations based on linear stability analysis for natural circulation loops have been carried out in [20-25]. They assumed homogeneous two-phase flow in their analyses and validated their predictions with the test data. Ishii and Zuber's[20] model was modified by Saha and Zuber [21] by taking into account the thermal non-equilibrium effect between the phases. They found that thermal non-equilibrium effect between the phases predicts a more stable system at low subcooling when compared with thermal equilibrium model. Furutera [22] found that the threshold of instability depends on the two-phase flow friction multiplier and the heat capacity in the subcooled boiling region .Lee and Lee [23] predicted the threshold of instability for Ledinegg and density-wave oscillations and showed that the density wave instability analysis is sufficient for the stability analysis of the two-phase flow system. Wang et al. [24] predicted the threshold of density wave instability for a two phase natural circulation loop and found that instability can appear at low as well as high power levels. The stability of density wave oscillations was found to decrease with decrease in (a) diameter of riser pipes and (b) system pressure. Nayak et.al. [25] performed the linear stability analysis to predict the threshold of instability for a

typical configuration of AHWR. They found that density wave threshold decreases with an increase in pressure and a decrease in inlet subcooling. They also found that existence of power distribution among the channels reduces the density wave instability if the channel exit quality is same.

Similar analysis has also been carried out by numerous investigators for forced convection boiling systems [26-29]. Sumida and Kawai [26] derived expressions for the hydraulic stability of the boiling channels in terms of flow impedance (defined by the dynamic response of pressure drop to the inlet flow). Based on these they classified the density wave instability into three main categories. Fukuda and Hasegawa [27] derived the mode of oscillations of parallel boiling channels from the characterstic equation. Taleyarkhan et. al [28] considered subcooled boiling, heat flux distribution in the channel, heater wall dynamics, slip flow and transient mixing of flow paths in their analysis. They found that subcooled boiling destablises the system and the system stability improves with ventilation between the channels. Clausse et. al. [29] found that for system having identical channels(a) system can oscillate with all the channels in-phase with the external loop (b) channels themselves oscillating out of phase maintain a constant flow rate in the loop. If the channels are different, complicated modes of oscillations involving coupling between the channels and loop can be formed.

1.3.2 Non linear analysis

Gurugenci et. al [30] developed a numerical code to generate limit cycles of pressure drop and density wave oscillations in a boiling up-flow system in a channel. Chatoorgoon et. al. [31] developed a computer code named as 'SPORTS' which solves the conservation equations numerically with minimum approximations, avoiding the use of property derivatives and matrix inversions. It also permits large and small time steps. This code can predict the limit cycle oscillations in both natural and forced circulation systems. Nigamutlin et. al [32] found that the heat transfer coefficient does not effect the threshold of stability while the thermal wall inertia has a strong stabilising effect.

1.3.3 Relevant to power reactors

Apart from all these fundamental studies cited above, other investigations exist on flow instabilities in power reactors. Krishnan and Gulshani [33] investigated the flow instabilities in the CANDU type PHWRs analytically and experimentally under two-phase parallel channel natural circulation conditions. They observed intermittent flow oscillations that occurred at low exit qualities caused by the periodic formation and expulsion of slug bubbles in the boiling channel: and, an oscillatory flow instability that occurs at high exit quality characterised by large amplitude sinusoidal type flow oscillations in the channel. Mochizuki [34] studied the instability behavior of the heavy water pressure tube type reactor in a simulated experimental facility both under natural and forced circulation conditions. Also, he carried out numerical investigations to clarify the various types of flow instabilities in the reactors.

All these studies clarified that the density-wave oscillation is the most common type of instability that can occur both in boiling natural circulation loops and in forced convection systems. Occurrence of this instability depends on the system pressure, channel inlet and outlet resistances, heater power and inlet subcooling conditions.

1.4 Objective of this thesis

For two-phase flow instabilities more accurate predictions are being sought. Earlier most of the codes developed for this purpose are based on the homogeneous model. In homogeneous model relative velocity between vapor and liquid has not been considered. However, in two phase flow there is always a relative motion between the liquid and vapor phase. In drift-flux model relative velocity between liquid and vapor is considered by taking into account the vapor drift velocity (V_{gj}) in the two-phase region. Earlier some codes based on drift flux model have been developed. But these codes are applicable to forced circulation loop only.

The objective of this thesis is to develop and validate the code based on drift flux model for prediction of stability limits and to study the effect of various parameters on stability.

CHAPTER 2

DRIFT FLUX MODEL

2.1 Modelling Of The Stability Behaviour of Two-phase Natural Circulation Loop

The mathematical model to be presented in this section has been applied to investigate the stability behaviour of single channel two-phase natural circulation loop such as shown in Fig. 2.1. Relative velocity between vapor and liquid is included by using the drift-flux model for two-phase region. In the analysis it is assumed that fluid is incompressible in the single phase region.

In the two-phase region we assume that

(a) the two-phases are in thermodynamic equilibrium,

(b) pressure profile in the loop is always in a quasi steady state i.e. $\frac{\partial p}{\partial t} = 0$,

(c) no carry-over or carry-under in steam drum (i.e. complete separation),

(d) complete instantaneous mixing of feed-water in the steam drum,

(e) heat losses in the pipings are negligible,

(f) properties don't change significantly along the loop, since Δp across the different components (riser, downcomer, cooler, heater and fittings) is << than system pressure,

(g) inlet subcooling ΔT_{sub} is constant for a particular power and feed-water temperature condition.

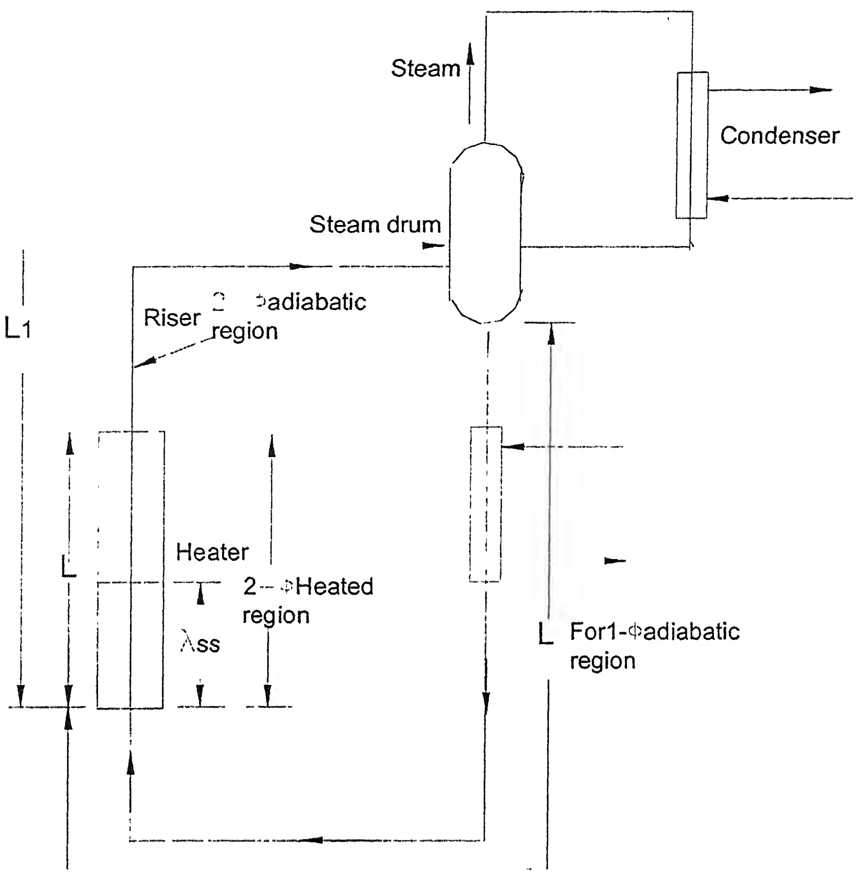


Fig 2.1 Schematic of natural circulation loop considered in the modelling

With these assumptions the conservation equations of mass, energy and momentum for one-dimensional two-phase flow are given by

Conservation of mass of mixture

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m V_m) = 0 \tag{1}$$

Conservation of mass of vapor phase

$$\frac{\partial}{\partial t}(\alpha \rho_g) + \frac{\partial}{\partial z}(\alpha \rho_g V_m) = \Gamma_g - \frac{\partial}{\partial z}\left(\alpha \frac{\rho_f \rho_g}{\rho_m} V_{gj}\right) \tag{2}$$

Conservation of energy of the mixture

$$\rho_m \frac{\partial h_m}{\partial t} + \rho_m V_m \frac{\partial h_m}{\partial z} = \frac{q_w'' P_h}{A_c} - \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m} \frac{\rho_f \rho_g}{\Delta \rho} V_{gj} h_{fg}\right) \tag{3}$$

Conservation of momentum of the mixture

$$-\frac{\partial p}{\partial z} = \rho_m \left(\frac{\partial V_m}{\partial t} + V_m \frac{\partial V_m}{\partial z}\right) + \frac{f_m}{2D} \rho_m V_m^2 + \rho_m g + \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_f \rho_g}{\rho_m} V_{gj}^2\right) \tag{4}$$

These equations have been taken from Belblidia et. al.[38]

The thermal equation of state gives the density as a function of saturation pressure

$$\rho_f = \rho_f(p_s); \qquad \rho_g = \rho_g(p_s) \qquad \rho_m = \alpha \rho_g + (1-\alpha)\rho_f \tag{5}$$

Caloric equation of state can be written as:

$$h_f = h_f(p_s); \quad h_g = h_g(p_s) \qquad h_m = \{\alpha \rho_g h_g + (1-\alpha)\rho_f h_f\}/\rho_m \tag{6}$$

The vapor drift velocity (V_{gj}) is velocity of vapor phase with respect to velocity of center of volume or volumetric flux density of the mixture.

The volumetric flux density of the mixture is given by

$$j = (1-\alpha)V_f + \alpha V_g \tag{A.5}$$

so vapour drift velocity is given by

$$V_{gj} = V_g - j$$

$$V_{gj} = V_g - (1-\alpha)V_f + \alpha V_g$$

$$V_{gj} = (1-\alpha)(V_g - V_f)$$

$$V_{gj} = (1-\alpha)(V_r) \tag{A.9}$$

Where V_r is relative velocity of vapour with respect to liquid phase.

So it accounts for the relative motion between two phases. If V_{gj} = 0.0, then present model will be equivalent to Homogeneous model.

2.2 Steady State Equation

The governing equations for the steady state conditions are obtained by dropping the time derivatives from equations (1) to (4).

2.2.1 For single-phase region

Conservation of mass equation becomes

$$\frac{\partial}{\partial z}(\rho_f V_f) = 0 \tag{7}$$

Conservation of energy equation becomes

$$\rho_f V_f \frac{\partial h_f}{\partial z} = \frac{q_w'' P_h}{A_c} \quad \text{for heated region} \quad (a)$$

$$= 0 \quad \text{for adiabatic region} \quad (b) \quad (8)$$

Conservation of momentum equation for the liquid phase

$$\frac{-\partial p}{\partial z} = \underbrace{\rho_f V_f \frac{\partial V_f}{\partial z}}_{\text{acceleration term}} + \underbrace{\frac{f_s}{2D}\rho_f V_f^{\ 2}}_{\text{viscousloss term}} + \underbrace{\rho_f g}_{\text{gravitaional term}} \tag{9}$$

2.2.2 For two-phase region

Conservation of mass of two-phase mixture

$$\frac{\partial}{\partial z}(\rho_m V_m) = 0 \tag{10}$$

Conservation of vapour mass for two-phase becomes

$$\frac{\partial}{\partial z}\left(\alpha \rho_g V_m\right) = \Gamma_g - \frac{\partial}{\partial z}\left(\frac{\alpha \rho_f \rho_g}{\rho_m} V_{gj}\right) \quad \text{for heated region} \quad \text{(a)}$$

$$= \frac{-\partial}{\partial z}\left(\frac{\alpha \rho_f \rho_g}{\rho_m} V_{gj}\right) \quad \text{for adiabatic region} \quad \text{(b)} \quad (11)$$

Which can be transformed to (refer equation B.11 to B.29)

$$\frac{C_k \partial \rho_m}{\partial z} = -\Omega_{ss}\rho_{mss} \quad \text{for heated region}$$

$$= 0 \quad \text{for adiabatic region} \quad (12)$$

Conservation of energy of the mixture

$$\rho_m V_m \frac{\partial h_m}{\partial z} = \frac{q_w^{"} P_h}{A_c} - \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m}\frac{\rho_f \rho_g}{\Delta \rho} V_{gj} h_{fg}\right) \text{ for heated region} \quad \text{(a)}$$

$$= \frac{-\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m}\frac{\rho_f \rho_g}{\Delta \rho} V_{gj} h_{fg}\right) \quad \text{for adiabatic region} \quad \text{(b)} \ (13)$$

$C_k = j + V_{gj}$ (This is an approximation for slug flow) (A.14)

$$j = V_m + \alpha \frac{\Delta \rho}{\rho_m} V_{gj} \tag{A.10}$$

$$\Omega_{ss} = \frac{\Gamma_g \Delta \rho}{\rho_f \rho_g} \quad \text{where} \quad \Gamma_g = \text{vapour generation rate} = \frac{q_w^{"} P_h}{A_c h_{fg}}$$

So $\Omega_{ss} = \dfrac{q''_w P_h}{A_c h_{fg}} \dfrac{\Delta\rho}{\rho_f \rho_g}$

Conservation of two-phase momentum

$$\frac{-\partial p}{\partial z} = \underbrace{\rho_m V_m \frac{\partial V_m}{\partial z}}_{\text{acceleration term}} + \underbrace{\frac{f_m}{zD}\rho_m V_m^2}_{\text{viscousloss term}} + \underbrace{\rho_m g}_{\text{gravitaional term}} + \underbrace{\frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g}\frac{\rho_f \rho_g}{\rho_m} V_{gj}^2\right)}_{\text{slip loss term}} \tag{14}$$

2.3 Steady state solution

2.3 (a) For the single phase region

Equations (7) to (9) for the single-phase region can be solved analytically and the expressions for pressure drop for single-phase region can be obtained as

$$(\Delta p)_{1-\phi} = \rho_f g \Delta Z + \frac{f_s \rho_f V_{fiss}{}^2 L}{2D} + \frac{K_l \rho_f V_{fiss}{}^2}{2} \quad \text{for adiabatic region} \quad \text{(a)}$$

$$= \rho_f g \lambda_{ss} + \frac{f_s \rho_f V_{fiss}{}^2 \lambda_{ss}}{2D} + \frac{K_l \rho_f V_{fiss}{}^2}{2} \quad \text{for heated region} \quad \text{(b)} \quad (15)$$

In equation (15) acceleration term is zero due to conservation of mass and incompressibility of fluid.

λ_{ss} = length of single phase region in heated channel

ΔZ = change in height (elevation difference)

2.3 (b) For the two phase region

Equations 10–14 can be solved analytically to obtain the pressure drop for 2-ϕ region.

(1) acceleration term= $\int_{\lambda ss}^{L} \rho_{mss} V_{mss} \dfrac{\partial V_{mss}}{\partial z} \partial z$

$$= \int_{\lambda ss}^{L} \rho_{mss} V_{mss} \partial V_{mss} \quad (\text{since } \rho_{mss} V_{mss} = \rho_{fiss} V_{fiss})$$

$$= \rho_{mss} V_{mss} \int_{\lambda ss}^{L} \partial V_{mss} = \rho_{mss} V_{mss} \left[V_{mss,L} - V_{mss,\lambda ss}\right]$$

(2) viscous loss term= $\int_{\lambda ss}^{L} \dfrac{f_m}{2D} \rho_{mss} V_{mssf}^2 dz$

$$= \int_{\lambda ss}^{L} \frac{f_m}{2D} \frac{\rho^2{}_{mss} V_{mss}^2}{\rho_{mss}} dz$$

Now since $\rho_{mss} V_{mss} = \rho_{fiss} V_{fiss}$

Viscous loss term= $\int_{\lambda ss}^{L} \frac{f_m}{2D} \frac{\rho^2{}_{fiss} V_{fiss}^2}{\rho_{mss}} dz = \frac{f_m}{2D} \rho^2{}_{fiss} V_{fiss}^2 \int_{\lambda ss}^{L} \frac{1}{\rho_{mss}} dz$

Now from equation $\rho_{mss} = \frac{\rho_{fiss} C_{kss,\lambda_{ss}}}{C_{kss}}$ (B.40)

$$= \frac{f_m}{2D} \rho^2{}_{fiss} V_{fiss}^2 \int_{\lambda ss}^{L} \frac{C_{kss}}{\rho_{fiss} C_{kss,\lambda ss}} dz$$

now dz= $\frac{dC_{kss}}{\Omega_{ss}}$ (from equation B.36)

so viscous loss term $= \frac{f_m}{2D} \frac{\rho_{fiss} V_{fiss}^2}{C_{kss,\lambda ss}} \int_{\lambda ss}^{L} \frac{C_{kss} dC_{kss}}{\Omega_{ss}} = \frac{f_m}{4D} \frac{\rho_{fiss} V_{fiss}^2}{\Omega_{SS} C_{kss,\lambda ss}} \left[C^2{}_{kss,L} - C^2{}_{kss,\lambda ss} \right]$

gravitational loss term= $\int_{\lambda ss}^{L} \rho_{mss} g dz$

using $\rho_{mss} = \frac{\rho_{fiss} C_{kss,\lambda_{ss}}}{C_{kss,L}}$,we get

$$= \int_{\lambda_{ss}}^{L} \frac{\rho_{fiss} C_{kss,\lambda_{ss}}}{C_{kss}} g dz = \rho_{fiss} g C_{kss,\lambda_{ss}} \int_{\lambda_{ss}}^{L} \frac{dz}{C_{kss}}$$

using dz= $\frac{dC_{kss}}{\Omega_{ss}}$ (B.36)

so gravitational loss term = $\rho_{fiss} g C_{kss,\lambda_{ss}} \int_{\lambda_{ss}}^{L} \frac{1}{C_{kss}} \frac{dC_{kss}}{\Omega_{ss}} = \frac{\rho_{fiss} g C_{kss,\lambda_{ss}}}{\Omega_{ss}} \left[\ln C_{kss} \right]_{C_{kss,\lambda ss}}^{C_{kss,L}}$

$$= \frac{\rho_{fiss} g C_{kss,\lambda_{ss}}}{\Omega_{ss}} \left[\ln C_{kss,L} - \ln C_{kss,\lambda ss} \right] = \frac{\rho_{fiss} g C_{kss,\lambda_{ss}}}{\Omega_{ss}} \left[\frac{\ln C_{kss,L}}{\ln C_{kss,\lambda ss}} \right]$$

(4) Slip loss term= $\int_{\lambda ss}^{L} \frac{\partial}{\partial z} \left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_f \rho_g}{\rho_m} V_{gj}^2 \right) \partial z$

$$= \left[\left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g} \frac{\rho_f \rho_g}{\rho_m} V_{gj}^2 \right) \right]_{\lambda ss}^{L} = \left(\frac{\rho_f - \rho_{mss,L}}{\rho_{mss,L} - \rho_g} \frac{\rho_f \rho_g}{\rho_m} V_{gj}^2 \right)$$

$$(\Delta p)_{2-\Phi} = \rho_{mss} V_{mss} \left[V_{mss,L} - V_{mss,\lambda ss}\right] + \frac{f_m}{4D} \frac{\rho_{fiss} V_{fiss}^2}{\Omega_{SS} C_{kss,\lambda ss}} \left[C^2{}_{kss,L} - C^2{}_{kss,\lambda ss}\right] + \frac{K_l \rho_m V_m{}^2}{2}$$

$$+ \frac{\rho_{fss} g C_{kss,\lambda_{ss}}}{\Omega_{ss}} \left[\frac{\ln C_{kss,L}}{\ln C_{kss,\lambda ss}}\right] + \left(\frac{\rho_f - \rho_{mss,L}}{\rho_{mss,L} - \rho_g} \frac{\rho_f \rho_g}{\rho_m} V_{gj}^2\right) \quad \text{for heated region} \quad (a)$$

$$\text{and } (\Delta p)_{2-\phi} = \rho_{mss} g\,\Delta z + \frac{f_m}{2D} \rho_{mss} V_{mss}^2 L + \frac{K_l \rho_{mss} V_{mss}{}^2}{2} \quad \text{for adiabatic region} \quad (b) \quad (16)$$

It is important to note that ΔZ is elevation difference in the direction of flow. If the flow is from higher to lower elevation then ΔZ will be with a negative sign.

now $(\Delta p)_{total} = \Sigma(\Delta p)_{1-\phi adiabatic} + \Sigma(\Delta p)_{1-\phi heated} + \Sigma(\Delta p)_{2-\phi heated} + \Sigma(\Delta p)_{2-\phi adiabatic}$ (17)

since it is close loop with no pump.

$(\Delta p)_{total=0}$ 0

So $\Sigma(\Delta p)_{1-\phi adiabatic} + \Sigma(\Delta p)_{1-\phi heated} + \Sigma(\Delta p)_{2-\phi heated} + \Sigma(\Delta p)_{2-\phi adiabatic} = 0.0$ (18)

For a assumed flow rate value of pressure drop can be calculated, since all other variables such as C_k, V_m, V_{fi} can be obtained. So equation (18) can be solved iteratively to get the steady state flow rate.

2.4 Linear Stability Analysis

2.4.1 Perturbation Variables

The set of conservation equations (1) to (4) are linearised by superimposing of small perturbations of V'_m, W', V'_{fi}, h'_f, ρ'_m, C'_k over the steady state such as follows:

$$V_{fi} = V_{fiss} + V'_{fi},\ h_f = h_{fss} + h'_f,\ \rho_m = \rho_{m,ss} + \rho'_m, V_m = V_{mss} + V'_m \quad C_k = C_{kss} + C'_k \quad (19)$$

where $V'_{fi} = \overline{V}_{fi} \varepsilon e^{st}, V'_m = \overline{V}_m \varepsilon e^{st}, h'_f = \overline{h}_f \varepsilon e^{st}\ \rho'_m = \overline{\rho}_m \varepsilon e^{st},\ Ck' = \overline{Ck}\, \varepsilon e^{st}$ (20)

$\overline{V}_{fi}, \overline{h}_f$, $\overline{V}_m, \overline{\rho}_m$, $\overline{C}_k$ are average of perturbed variables and s is the stability parameter.

The governing (conservation) equations are linearised and solved analytically. Kinematics of the flow is solved first. Then the momentum equation is integrated to obtain the response in total pressure drop perturbation to an inlet flow rate perturbation.

2.4.2 Kinematics of the flow

2.4.2 (a) For the single phase heated region

The perturbed energy conservation equation for single phase heated region can be written as (refer equation B.1 to B.5)

$$\frac{\partial h'_f}{\partial z} + \frac{s h'_f}{V_{fiss}} = \frac{-q''_w P_h V'_{fi}}{\rho_f V_{fiss}^2 A} \quad (21)$$

Solution of equation (21) is

$$h'_{f,\lambda ss} = \frac{-q''_w P_h}{\rho_f A_c s} \frac{V'_{fi}}{V_{fiss}} \left\{ 1 - e^{-\left(\frac{s\lambda ss}{v_{fiss}}\right)} \right\} \text{(refer equation B.6 to B.10)} \tag{22}$$

from equation (8) $$\frac{\partial h_{fss}}{\partial z} = -\frac{q''_w P_h}{\rho_f V_{fiss} A_c} \tag{23}$$

For a uniform axial heat flux, the enthalpy increases linearly with Z. A positive perturbation in enthalpy at the boiling boundary causes a negative perturbation in the boiling boundary. Therefore, this can be expressed as

$$\frac{\partial h_{fss,\lambda ss}}{\partial z} = \frac{q''_w P_h}{\rho_f A_c V_{fiss}} = \frac{-h'_{f,\lambda ss}}{\lambda'}$$

$$\lambda' = -\frac{\rho_f A_c V_{fiss\,h}}{q''_w P_h} h'_{f,\lambda ss}$$

or $$\lambda' = -\frac{\rho_f A_c V_{fiss\,h}}{q''_w P_h} \frac{-q''_w P_h}{\rho_f A_c s} \frac{V'_{fi}}{V_{fiss}} \left\{ 1 - e^{-\left(\frac{s\lambda ss}{v_{fiss}}\right)} \right\}$$

hence $$\lambda' = \frac{1 - \exp^{-\tau_1(s)}}{s} V'_{fi} \tag{24}$$

where $\tau_1 = \lambda_{ss}/V_{fiss}$ is residence time of fluid in single phase region

2.4.2(b)For 2-Φ region in the core

Volumetric flux equation (refer equation B.11 to equation B.20)

$$\frac{\partial j}{\partial z} = \Omega$$

Density propagation equation(refer equationB.21 to equation B.28)

$$\frac{\partial \rho_m}{\partial t} + C_k \frac{\partial \rho_m}{\partial z} = -\rho_m \Omega$$

solution of volumetric flux equation is

$$j = V_{fi}(t) + \Omega\{Z - \lambda(t)\} \tag{25}$$

where $\lambda(t)$ is height of boiling boundary from the inlet of heated channel.

The perturbed portion of kinematic velocity can be written as

Since $C_k = V_{fi} + V_{gj}$

From equation (25) $C_k = V_{fi}(t) + \Omega\{Z - \lambda(t)\} + V_{gj}$

On perturbing both sides, we get

$$Ck_{ss} + Ck' = V_{fiss} + V'_{fi} + \Omega\{Z - \lambda_{ss} - \lambda'\} + V_{gj}$$

So $$C'_k = V'_{fi} - \Omega\lambda'$$

$$C'_k = V'_{fi}\{1 - \Omega\Gamma_1(s)\}$$

$$C'_k = \Gamma_2(s)V'_{fi} \tag{26}$$

The perturbed density propagation equation for core can be written as

(Refer equation B.28 to equation B.34)

$$\frac{\partial\rho'_m}{\partial z} + \frac{s + \Omega_{ss}}{Ck_{ss}}\rho'_m = \frac{C'_k\Omega_{ss}\rho_{mss}}{Ck_{ss}^2} \tag{B.34}$$

This can be integrated from λ_{ss} to Z using the fact that a positive perturbation in the boiling boundary causes a negative perturbation in mixture density. To a first approximation it can be written as

$$\frac{\partial}{\partial z}(\rho_{mss}) = -\frac{\Omega_{ss}\rho_f}{C_{kss,\lambda ss}} = -\frac{\rho'_{m,\lambda ss}}{\lambda'} \tag{27}$$

$$\rho'_{m,\lambda ss} = \frac{\Omega_{ss}\rho_f}{Ck_{ss,\lambda ss}}\lambda'$$

Thus, the solution for the mixture density is as follows(refer equation B.35 to B.48):

$$\rho'_m = \rho_f\left(\frac{\Omega_{ss}}{s - \Omega_{ss}}\right)\left(\frac{C_{kss,\lambda ss}}{C_{kss}}\right)^2\frac{1}{C_{kss,\lambda ss}}C'_k$$

$$+\left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s - \Omega_{ss}}C'_k\right)\left(\frac{\rho_f}{C_{kss,\lambda ss}}\right)\left(\frac{C_{kss,\lambda ss}}{C_{kss,L}}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} \tag{28}$$

On perturbing equation (B.17) and linearising, the solutions for the mixture velocity perturbations are obtained.(refer equation B.49 to B.51)

$$V'_m = \Gamma_2(s)V'_{fi} + \frac{V_{gj}}{\rho_f}\left(\frac{Ck_{ss}}{Ck_{ss,\lambda ss}}\right)^2\rho'_m \tag{29}$$

2.4.2(c) For unheated riser section

Since the riser is unheated, $\Omega = 0.0$

Volumetric flux equation

$$\frac{\mathrm{dj}}{\mathrm{dz}} = 0 \tag{30}$$

On perturbing above equation and solving

$$C_k' = j' = \text{constant} \tag{31}$$

density propagation equation

$$\frac{\partial \rho_m}{\partial t} + C_k \frac{\partial \rho_m}{\partial z} = 0 \tag{B.52}$$

On perturbing equation (B.52) and linearising(refer equation B.52 to B.55)

$$s\rho_m' + C_{kss} \frac{\partial \rho_m'}{\partial z} = 0 \tag{B.36}$$

On solving equation (37) density perturbation is obtained:

$$\rho_m' = \rho_{m,L}' e^{-s(z-L)/C_{kss,L}} \tag{B.55}$$

On perturbing equation (B.17) and solving for velocity perturbation .(refer equation B.58 to B.61)

$$V_m' = C_k' + \rho_f V_{gj} \rho_{m,L}' e^{-s(z-L/C_{kss\ L})} \tag{B.61}$$

2.4.3Dynamics of the Flow

By perturbing equation (4) perturbed momentum equation is obtained. Now this equations is integrated to obtain the perturbed pressure drop .

$$\frac{\Delta p_{total}'}{w_{1-\Phi}'} = \frac{\Delta p_g'}{w_{1-\Phi}'} + \frac{\Delta p_i'}{w_{1-\Phi}'} + \frac{\Delta p_a'}{w_{1-\Phi}'} + \frac{\Delta p_s'}{w_{1-\Phi}'} + \frac{\Delta p_l'}{w_{1-\Phi}'} + \frac{\Delta p_f'}{w_{1-\Phi}'} \tag{32}$$

where $w_{1-\Phi}' = \rho_f A V_{fi}'$ (33)

2.4.3(a)For single phase adiabatic region

(i) Inertia component (refer equation B.79 to B.80)

$$\frac{\Delta p_i'}{w_{1-\phi}'} = \frac{sL}{A}$$

(ii) Acceleration component (refer equation B.65 to B.68)

$$\frac{\Delta p_a'}{w_{1-\phi}'} = 0.0$$

(iii) Frictional component (refer equation B.69 to B.72)

$$\frac{\Delta p_f'}{w_{1-\phi}'} = \frac{f_s V_{fiss} L}{DA}$$

(iv) Loss component (refer equation B.73 to B.76)

$$\frac{\Delta p'_l}{w'_{1-\phi}} = \frac{2K_l V_{fiss}}{A}$$

(v) Gravitational term (refer equation B.62 to B.64)

$$\frac{\Delta p'_g}{w'_{1-\phi}} = 0.0$$

(vi) Slip component (refer equation B.77 to B.78)

$$\frac{\Delta p'_s}{w'_{1-\phi}} = 0.0$$

2.4.3 (b) For single-phase heated region

(i)Inertia component (refer equation B.95)

$$\frac{\Delta p'_i}{w'_{1-\phi}} = \frac{sL}{A_c}$$

(ii) Acceleration component (refer equation B.89 to B.90)

$$\frac{\Delta p'_a}{w'_{1-\phi}} = 0.0$$

(iii) Frictional component (refer equation B.86 to B.88)

$$\frac{\Delta p'_f}{w'_{1-\phi}} = \frac{f_s V_{fiss} L}{DA_C} + \frac{f_s V^2{}_{fiss} \Gamma_1(s)}{2DA_c}$$

(iv) Loss component (refer equation B.91 to B.92)

$$\frac{\Delta p'_l}{w'_{1-\phi}} = \frac{2K_l V_{fiss}}{A_c}$$

(v) Gravitational component (refer equation B.81 to B.85)

$$\frac{\Delta p'_g}{w'_{1-\phi}} = \frac{g\Gamma_1(s)}{A_c}$$

(vi) Slip component (refer equation B.93 to B.94)

$$\frac{\Delta p'_s}{w'_{1-\phi}} = 0.0$$

2.4.3 (c) For 2-phase heated region

For two-phase region following equations are used frequently

$$\frac{\rho'_{m,L}}{w'_{1-\Phi}}=\left(\frac{\Omega_{ss}}{S-\Omega_{ss}}\right)\left(\frac{Ck_{ss,\lambda ss}}{Ck_{ss,L}}\right)^{2}\frac{1}{Ck_{ss,\lambda ss}}\frac{\Gamma_s(s)}{A_c}$$

$$+\frac{1}{A_C}\left(\Omega_{ss}\Gamma_1(s)-\frac{\Omega_{ss}}{s-\Omega_{ss}}\Gamma_2(s)\right)\left(\frac{1}{C_{kss,\lambda ss}}\right)\left(\frac{C_{kss,\lambda ss}}{C_{kss,L}}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} \tag{34}$$

$$\frac{V'_{m,L}}{w'_{1-\Phi}}=\frac{\Gamma_2(s)}{\rho_f A}+\frac{V_{gj}}{\rho_f}\left(\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right)^{2}\frac{\rho'_{m,L}}{w'_{1-\Phi}} \tag{35}$$

(i)Inertia component (refer equation B.122 to B.126)

$$\frac{\Delta p'_i}{w'_{1-\phi}}=\left(\frac{s}{A_c}\right)\lambda_{ss}+\frac{C_{kss,\lambda ss}s\Gamma_2(s)}{\Omega_{ss}A_c}\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}+\left(\frac{V_{gj}s}{A_c}\frac{\Gamma_2(s)}{S-\Omega_{ss}}\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right)$$

$$+\left[\frac{V_{gj}s}{A_c}\left(\Gamma_1(s)\Omega_{ss}-\frac{\Gamma_2(s)\Omega_{ss}}{s-\Omega_{ss}}\right)\frac{1}{\Omega_{ss}-s}\left\{\left(\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}-1.0\right\}\right]$$

(ii)Acceleration component (refer equation B.127 to B.143)

$$\frac{\Delta p'_a}{w'_{1-\Phi}}=-\left(1-\frac{V_{gj}}{C_{kss,\lambda ss}}\right)\left[\frac{s}{s-\Omega_{ss}}\frac{\Gamma_2(s)}{A_c}C_{kss,\lambda ss}\left\{\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right\}\right.$$

$$\left.+\frac{s}{A_c}\left\{\Gamma_1(s)-\frac{\Gamma_2(s)}{s-\Omega_{ss}}\right\}\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}-\left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\right]$$

$$-\frac{V_{fiss}^2}{A_c}\left(\frac{s+\Omega_{ss}}{C^2{}_{kss,\lambda ss}}\right)\left[\frac{1}{s-\Omega_{ss}}C_{kss,\lambda ss}\Gamma_2(s)\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right.$$

$$\left.+\left(\Omega_{ss}\Gamma_1(s)-\frac{\Omega_{ss}}{s-\Omega_{ss}}\Gamma_2(s)\right)\frac{C_k^{\frac{s}{\Omega_{ss}}}}{\Omega_{ss}-s}\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}-\left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\right]$$

$$-\frac{2V_{fiss}\Gamma_2(s)}{A_c}\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}-\frac{2V_{fiss}V_{gj}}{C^2_{kss,\lambda ss}}\frac{X}{w'_{1-\Phi}}$$

$$\frac{X}{w'_{1-\Phi}}=\left(\frac{\Omega_{ss}}{s-\Omega_{ss}}\frac{C_{kss,\lambda_{ss}}\Gamma_2(s)}{A_c}\ln\frac{C_{kss,\lambda ss}}{C_{kss,L}}\right)+\left[\left(\Omega_{ss}\Gamma_1(s)-\frac{\Omega_{ss}}{s-\Omega_{ss}}\Gamma_2(s)\right)\frac{\left(C_{kss,\lambda_{ss}}\right)^{\frac{s}{\Omega_{ss}}}}{A_c}\right.$$

$$\left.\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{SS}}{\Omega_{SS}-s}}-\left(C_{kss,\lambda_{SS}}\right)^{\frac{\Omega_{SS}}{\Omega_{SS}-s}}\right\}\right]$$

(iii)Frictional component (refer equation B.101 to B.117)

$$\frac{\Delta p'_{f2-\phi}}{w'_{1-\phi}}=\frac{f_m}{DA_c}V_{fiss}\Gamma_2(s)(L-\lambda_{ss})+\frac{f_m}{\rho_f DA_c}\frac{V_{gj}}{C^2_{kss,\lambda ss}}V_{fiss}\left(\frac{F}{V'_{fi}}\right)$$

$$+\frac{f_m}{2D\rho_f A_c}\left(1-\frac{V_{gj}}{C_{kss,\lambda ss}}\right)^2\left(\frac{F}{V'_{fi}}\right)-\frac{f_s V^2{}_{fiss}\Gamma_1(s)}{2DA_c}$$

Where $\frac{F}{V'_{fi}}=\frac{\rho_f}{s-\Omega_{ss}}C_{kss,\lambda ss}\Gamma_2(s)\left(C_{kss,L}-C_{kss,\lambda ss}\right)$

$$+\left(\Gamma_1(s)-\frac{\Gamma_2(s)}{s-\Omega_{ss}}\right)\rho_f\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\frac{\Omega_{ss}}{2\Omega_{ss}-s}\right)\left[\left(\frac{\Omega_{ss}}{s-\Omega_{ss}}\right)^{\frac{2\Omega_{ss}-s}{\Omega_{ss}}}-\left(C_{kss,\lambda ss}\right)^{\frac{2\Omega_{ss}-s}{\Omega_{ss}}}\right]$$

(iv) Local loss component (refer equation B.144 to B.147)

$$\frac{\Delta p'_l}{w'_{1-\phi}}=2k_l\rho_{mss,L}V_{mss,L}\frac{V'_{m,L}}{w'_{1-\phi}}+\left(K_e\frac{\rho'_{m,L}}{w'_{1-\phi}}V^2_{mss,L}\right)$$

(v) Gravitational component (refer equation B.96 to B.100)

$$\frac{\Delta p'_g}{w'_{1-\phi}}=\frac{-\Omega_{ss}}{sA_c}\left[\Gamma_1(s)-\Gamma_2(s)\left(\frac{1}{s-\Omega_{ss}}\right)\right]\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left[\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}-\left(C_{kss,L}\right)^{\frac{-s}{\Omega_{ss}}}\right]g$$

$$+\frac{1}{s-\Omega_{ss}}\frac{C_{kss,\lambda ss}}{A_c}\Gamma_2(s)\left(\frac{1}{C_{kss,\lambda ss}}-\frac{1}{C_{kss,L}}\right)g-\frac{g\Gamma_1(s)}{A_c}$$

(vi) Slip component (refer equation B.118 to B.121)

$$\frac{\Delta p'_s}{w'_{1-\phi}}=\frac{\rho'_{m,L}}{w'_{1-\phi}}\frac{\rho_f\;\rho_f V_{gj}{}^2}{\rho^2{}_{mss}}\left(1-\frac{2\rho_f}{\rho_{mss,L}}\right)$$

2.4.3 (d) For riser section

Following equations are frequently used. Value is given by

$$\frac{\rho'_{m,L_1}}{w'_{1-\Phi}} = \frac{\rho'_{m,L}}{w'_{1-\Phi}} e^{-s(L_1-L)/C_{kss,L}} \tag{36}$$

$$\frac{V'_{m,l_1}}{w'_{1-\Phi}} = \frac{\Gamma_2(s)}{\rho_f A_c} + \rho_f V_{gj} \frac{\rho'_{m,L}}{w'_{1-\Phi}} e^{-s(L_1-L/C_{kss,L})} \tag{37}$$

(i) Inertia component(refer equation B.161 to B.164)

$$\frac{\Delta p'_i}{w'_{1-\phi}} = \frac{s\rho_{mss}\Gamma_2(s)}{\rho_f A_c}(L_1 - L) + \left(\frac{\rho_f V_{gj}}{\rho_{mss,L}}\right)\left(\frac{\rho'_{m,L}}{w'_{1-\phi}}\right)\left(-C_{kss,L}\right)\left(\exp^{-\frac{s(L_1-L)}{C_{kss,L}}} - 1.0\right)$$

(ii) Acceleration component (refer equation A1.165 to B.172)

$$\frac{\Delta p'_a}{w'_{1-\phi}} = \left\{e^{-\frac{s(L_1-L)}{C_{kss,L}}} - 1\right\}\left(\frac{\rho'_{m,L}}{w'_{1-\phi}} C_{kss,L} V_{mss,L_1} - \frac{\rho'_{m,L}}{w'_{1-\phi}} V^2_{mss,L}\right)$$

(iii) Frictional component (refer equation B.152 to 155)

$$\frac{\Delta p'_f}{w'_{1-\phi}} = \frac{f_m}{D}\rho_{mss}V_{mss}\left[\frac{\Gamma_2(s)(L_1-L)}{\rho_f A_c} + \rho_f V_{gj}\frac{\rho'_{m,L}}{w'_{1-\phi}}\left(\frac{-C_{kss}}{s}\right)\left\{e\frac{-s(L_1-L)}{C_{kss,L}} - 1.0\right\}\right]$$

$$+\left[\frac{f_m}{2D}V^2_{mss}\left(\frac{\rho'_{m,L}}{w'_{1-\phi}}\right)\left(\frac{-C_{kss}}{s}\right)\left\{e\frac{-s(L_1-L)}{C_{kss}} - 1.0\right\}\right]$$

(iv) Loss component (refer equation B.173 to B.174)

$$\frac{\Delta p'_l}{w'_{1-\phi}} = 2K_l\rho_{mss}V_{mss}\frac{V'_{m,L_1}}{w'_{1-\phi}} + K_l\frac{\rho_{m,L_1}}{w'_{1-\phi}}V^2_{mss}$$

(v) Gravitational component(refer equation B.148 to B.151)

$$\frac{\Delta p'_g}{w'_{1-\phi}} = \left(\frac{\rho'_{m,L}}{w'_{1-\phi}}\right)g\left(\frac{-C_{kss,L}}{s}\right)\left\{\exp\frac{-s(L_1-L)}{C_{kss,L}} - 1.0\right\}$$

(vi) Slip component (refer equation B.156 to B.160)

$$\frac{\Delta p'_s}{w'_{1-\phi}} = \frac{-2V^2_{gj}\rho_f^2\rho_g}{\rho^3_{mss}}\left(\frac{\rho'_{m,L_1}}{w'_{1-\phi}} - \frac{\rho'_{m,L}}{w'_{1-\phi}}\right) + \frac{\rho_f\rho_g V^2_{gj}}{\rho^2_{mss}}\left(\frac{\rho'_{m,L_1}}{w'_{1-\phi}} - \frac{\rho'_{m,L}}{w'_{1-\phi}}\right)$$

Now $\Delta p'_{total}$ for the loop can be calculated.

$\Delta p_{total} = (\Delta p_{ss})_{total} + (\Delta p')_{total} = 0.0$ for a closed natural circulation loop

but $(\Delta p_{ss})_{total} = 0.0$ from equation (18)

so $(\Delta p')_{total} = 0.0$

but $(\Delta p')_{total} = \Sigma(\Delta p')_{1-\phi adiabatic} + \Sigma(\Delta p')_{1-\phi heated} + \Sigma(\Delta p')_{2-\phi heated} + \Sigma(\Delta p')_{2-\phi adiabatic} = 0.0$ (38)

Equation (38) is the characteristic equation. It is solved for the stability parameter (complex no.).If the s is the root of the characteristic equation given by

$s = \eta + j\varpi$

Where η is the real part and ϖ is the imaginary part of the root. Then the system is considered to be stable if all $\eta<0$, and unstable if any $\eta>0$.at the neutral threshold point at least one of the $\eta=0$.

Decay ratio defined as the ratio of the successive amplitude of impulses can be obtained as:

$$DR = \exp^{\frac{2\pi\eta}{|\varpi|}} \quad (39)$$

The decay ratio indicates the stability margin, which the system can have at, any operating condition. If decay ratio is less than one then the system is stable. If it is more than one the system is considered unstable. If it is equal to one then the system is at threshold of stability. In this condition system is called neutrally stable.

CHAPTER 3

MODEL VALIDATION

3.1 Experimental Loops

It is required to validate the theoretical model discussed earlier in chapter 2. For this purpose the code TINFLOS-D has been developed and validated with experimental data obtained from two natural circulation loops in BARC- (a) HPNCL (b) Apsara.The first experimental loop HPNCL is in Hall No. 7 of BARC.The second natural circulation loop Apsara is locted near Apsara reactor in BARC. These loops along with the geometry are shown in Fig3.1 and Fig 3.2 respectively.

Code TINFLOS-D has also been validated with two other natural circulation loops reported in literature. One experimental result obtained from Chexal and Bergles [3] and other experimental data obtained from M. Furutera [21]. These loops and their geometry are shown on Fig 3.3 and Fig 3.4 respectively

3.2 Validation

Fig. 3.5 shows the comparison of predicted mass flow rate with experimental mass flow rate for HPNCL. It can be seen that prediction are in good agreement with test data. Fig 3.6 and Fig 3.7 also shows the threshold power of stability for experimental data and TINFLOS-D at 1°C and 2.5°C subcooling respectively for the HPNCL. Table 3.7 also shows that predictions are in agreement with the experimental results for HPNCL.

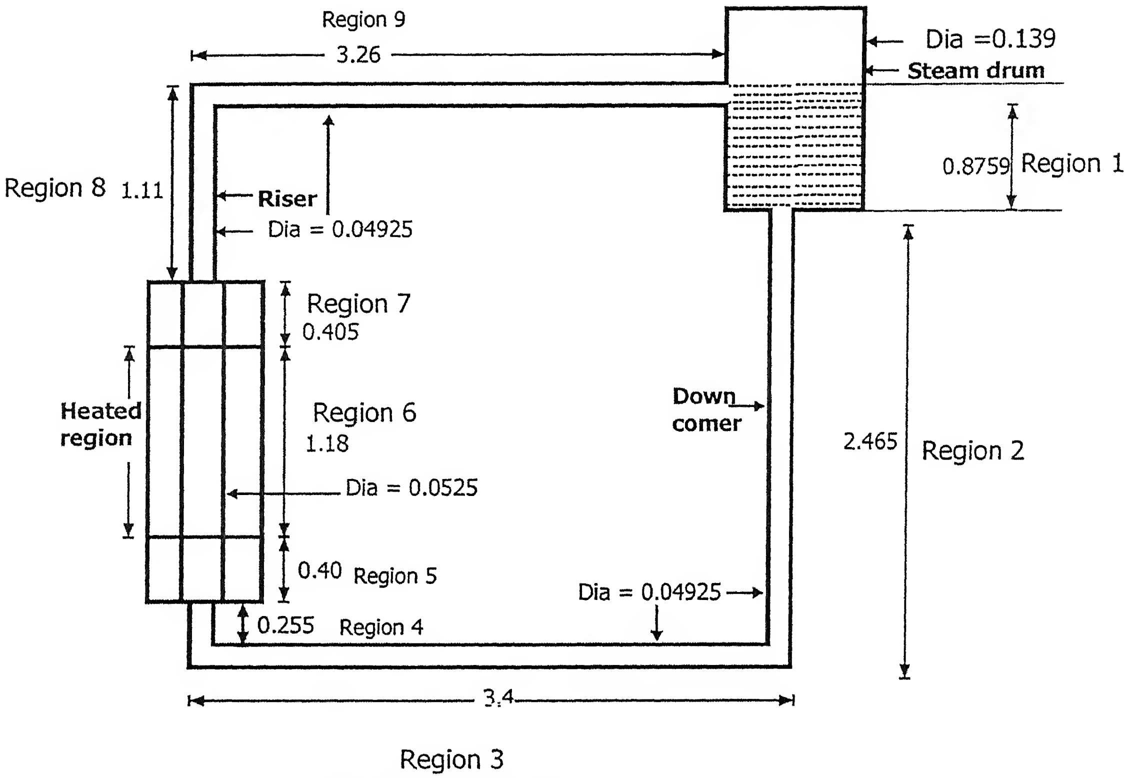
Table3.8 shows the validation of predicted data with experimental data for Apsara loop. It can be seen that predictions are in good agreement with the experimental results. Fig. 3.8 and Fig.3.10 show validation of predicted data with experimental data for Furutera's loop and Bergles loop respectively. It can be clearly seen from these figures that predicted data are in good agreement with the test data.

In Furutera's loop there are two spherical buffer volumes as shown in Fig 3.3.these spherical volumes will act as dampener for oscillations. But in present analysis the effect of these buffer volumes has not been considered, and these buffer volumes has been considered as the pipe of constant diameter 0.0215 meter. Diameter of Steam separator is not available in literature. So it has been considered to be of very large diameter, so that area ratio $A_r = \frac{A_1}{A_2} \approx 0$. Value of throttling loss coefficient and the orifice loss coefficient is not available. So hit and trial procedure has been adopted for proposed throttling and orifice loss coefficient. Fig. 3.9 shows the comparison of experimental results with the predictions by 7 models .Fig. 3.9 and table 3.6 are reported in Furutera et. al.[21].It is given in the literature that the prediction by model DST-1 is best followed by DST-0 and DST-6.on comparing the errors (table) we find that Prediction by DST-1 is most accurate followed by TINFLOS-D(V_{gj_slug}), DST-6, TINFLOS-D(V_{gj}=0.0),DST-0,DST4,DST-2,DST5 and DST-3 respectively.DST-1 is predicting better than the TINFLOS-D(V_{gj_slug}).It is due to the fact that in DST-1 subcooled boiling is considered, while in TINFLOS-D(V_{gj_slug}) subcooled boiling has been neglected.

In Bergles loop there is a scope for improvement in simulation, since subcooled boiling has not been considered. Also in the Bergles loop value of inlet valve loss coefficient and orifice loss coefficient is not available in the literature. So hit and trial procedure has been adopted for the proposed inlet valve and orifice loss coefficient.

Table 3.1 Errors associated with predictions

Model	sum of error square/N
DST-0	291.711
DST-1	11.86
DST-6	238.425
TINFLOS-D(V_{gj_slug})	44.134
TINFLOS-D(V_{gj}=0)	259.87



All dimensions in meter

Fig 3.1 HPNCL

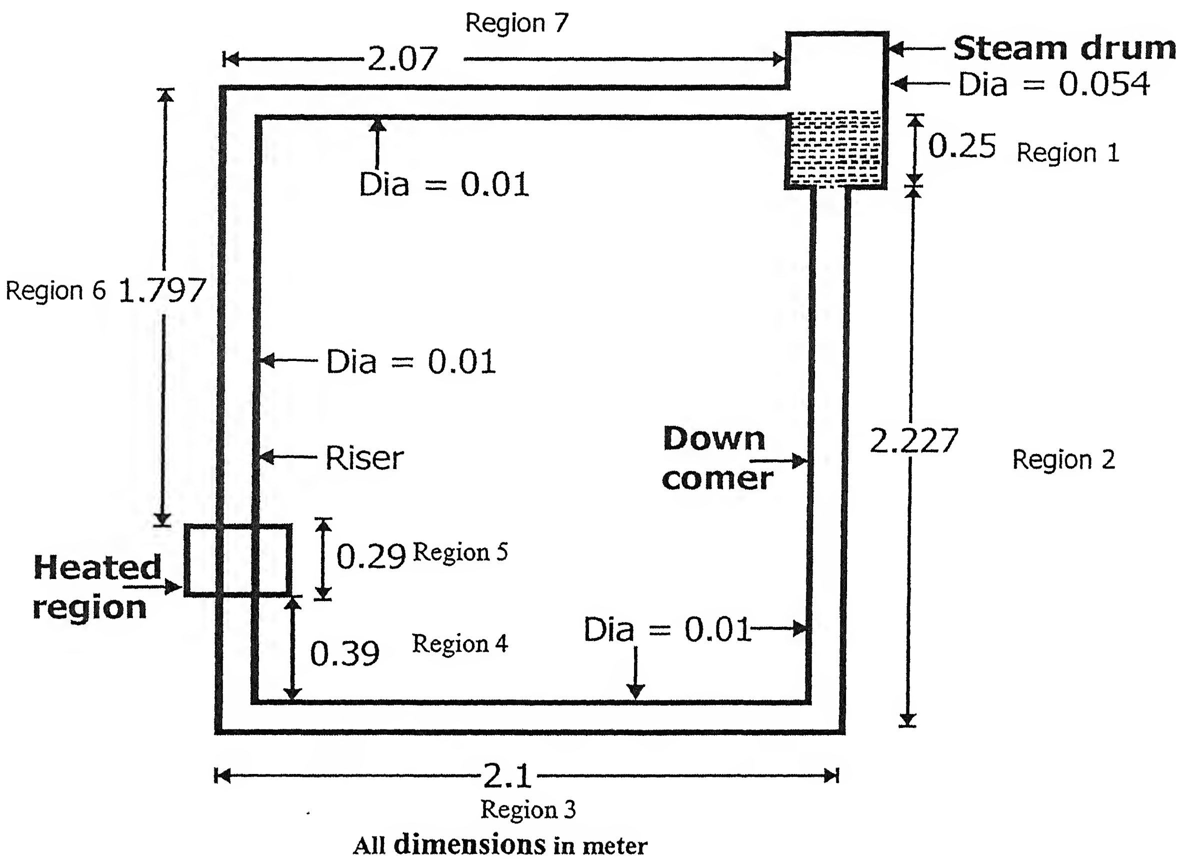
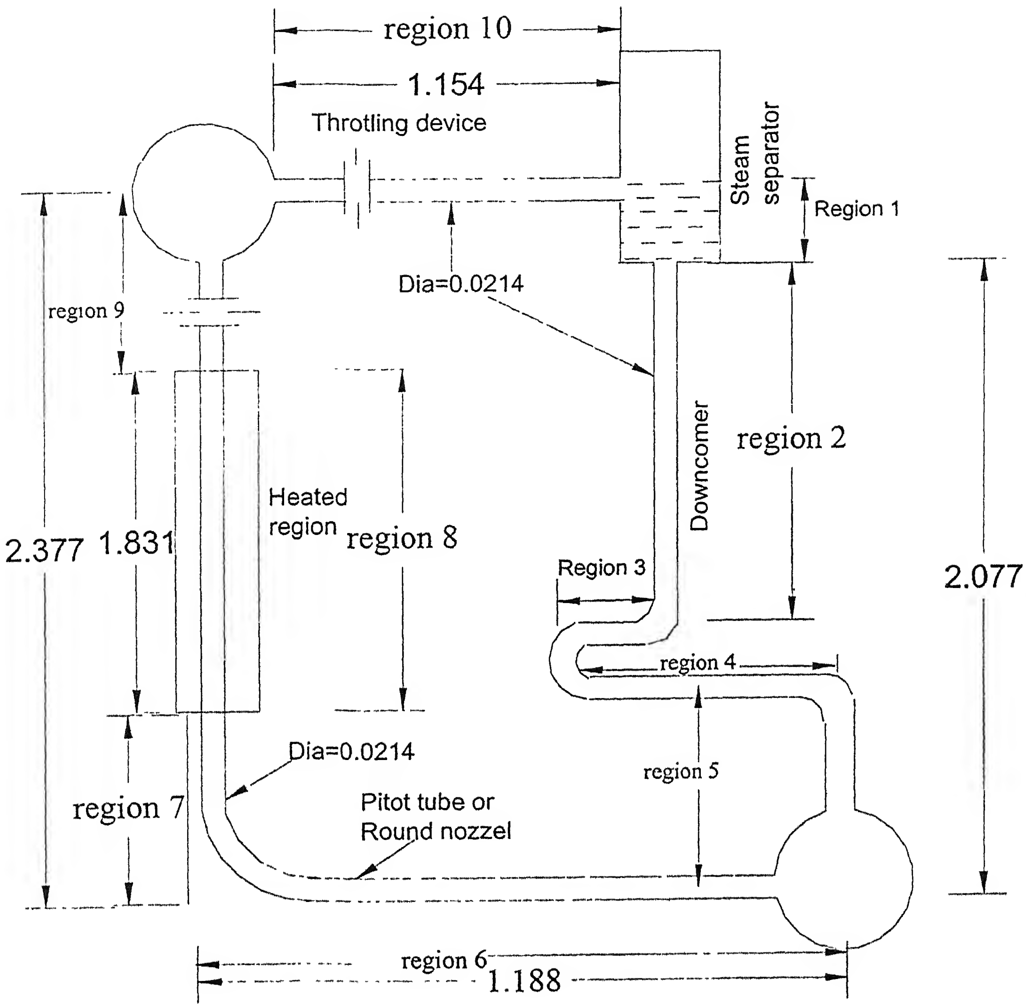
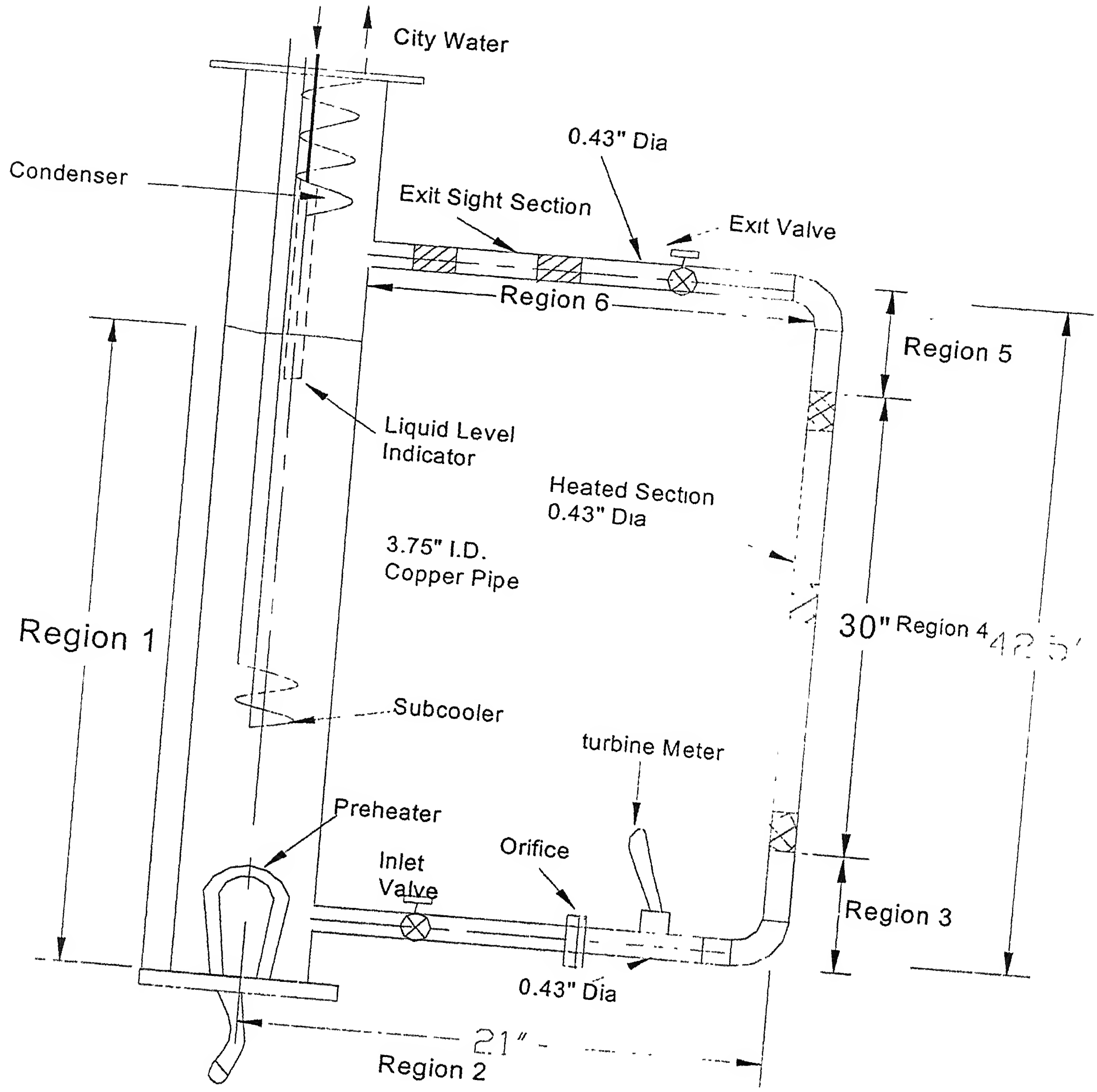


Fig 3.2 Apsara loop



All dimensions in meter

Fig 3.3 Furutera's loop



All dimensions in inch

Fig 3.4 Bergles loop

Table 3.2: Data for HPNCL Loop

	Regions								
	1	2	3	4	5	6(core)	7	8	9
Δz	-0.8759	-2.456	0.0	0.255	0.40	1.18	0.405	1.11	0.0
L	0.8759	2.456	3.4	0.255	0.40	1.18	0.405	1.11	3.26
K_{lexit}	0.4526	1.185	1.185	0.0144	0.0	0.0	0.102	1.185	0.767

Table 3.3: Data for Apsara Loop

	Regions						
	1	2	3	4	5(core)	6	7
Δz	-0.25	-2.227	0.0	0.39	0.29	1.797	0.0
L	0.25	2.227	2.10	0.39	0.29	1.797	2.07
K_{lexit}	0.4936	1.185	1.185	0.0	0.0	1.185	0.9663

Table 3.4: Data for Furutera's Loop

	Regions									
	1	2	3	4	5	6	7	8(core)	9	10
Δz	-0.30	-1.7655	0.0	0.0	-0.3115	0.0	0.3115	1.831	0.2359	0.0
L	0.30	1.7655	0.01	0.034	0.3115	1.188	0.3115	1.831	0.2359	1.154
$K_{l\,exit}$	0.50	1.185	2.37	1.185	1.185	1.185	0.0	0.0	1. 185	1.0

Table 3.5: Data for Bergles Loop

	Regions					
	1	2	3	4(core)	5	6
Δz	-1.0795	0.0	0.15875	0.762	0.15875	0.0
L	1.0795	0.5334	0.15875	0.762	0.15875	0.5334
K_{lexit}	0.495	1.185	0.0	0.0	1.185	0.97

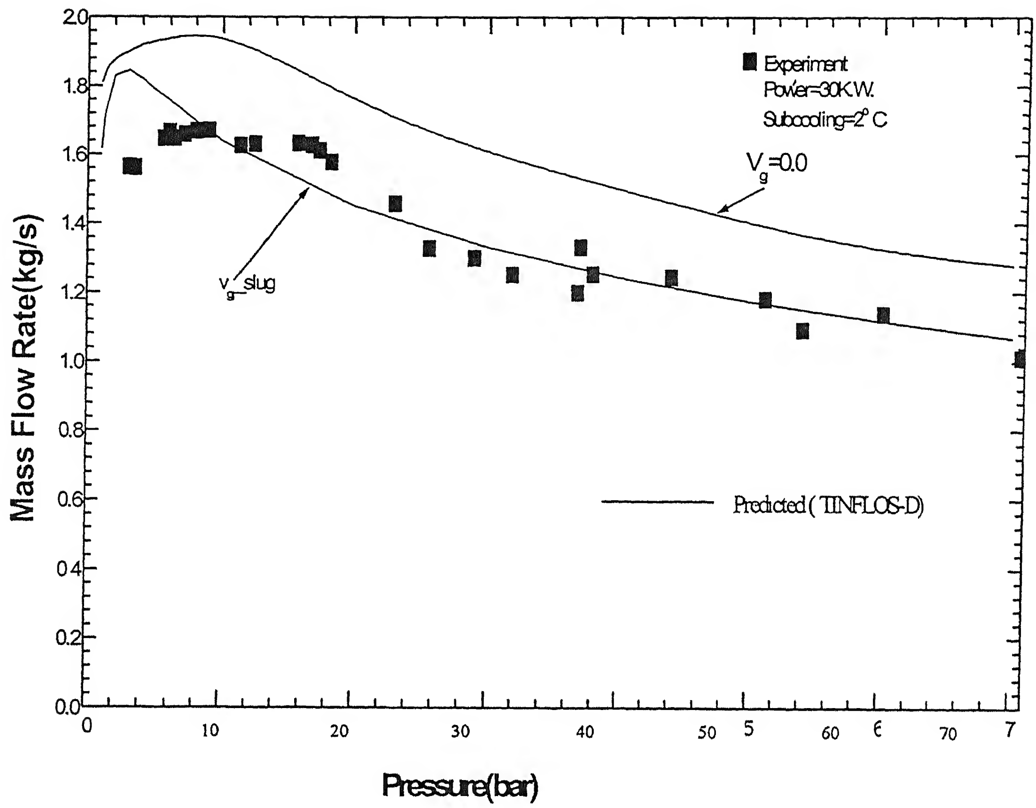


Fig 3.5 Comparison of predicted mass flow rate with experimental mass flow rate for HPNCL under steady state condition

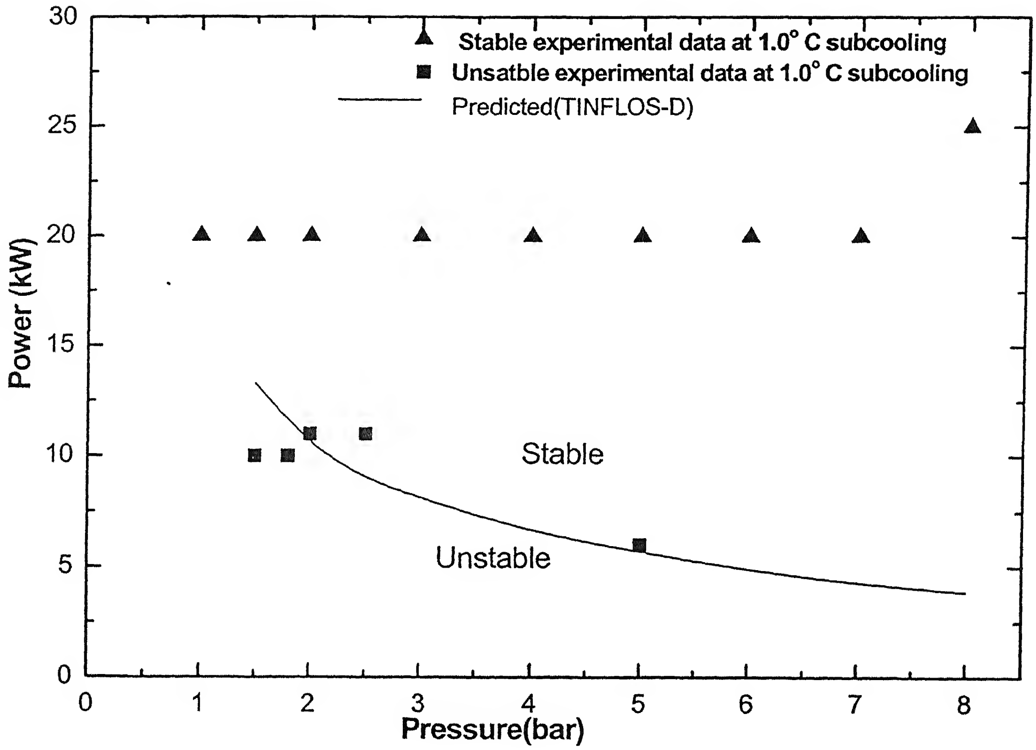


Fig 3.6 Threshold power of stability–prediction and experimental data for HPNCL

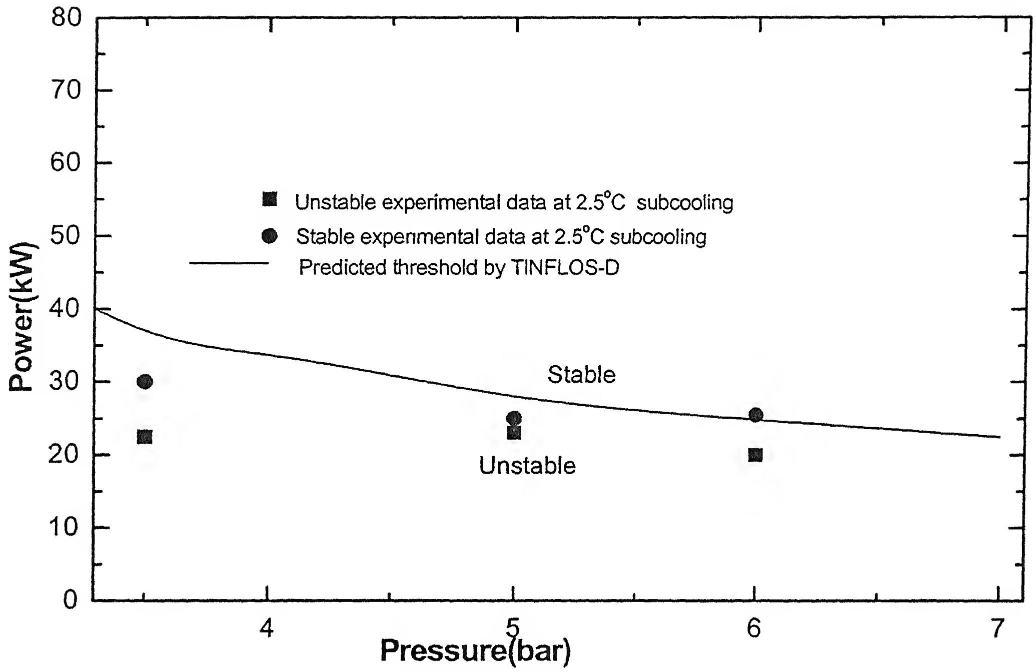


Fig 3.7 Threshold power of stability– prediction and experimental data for HPNCL

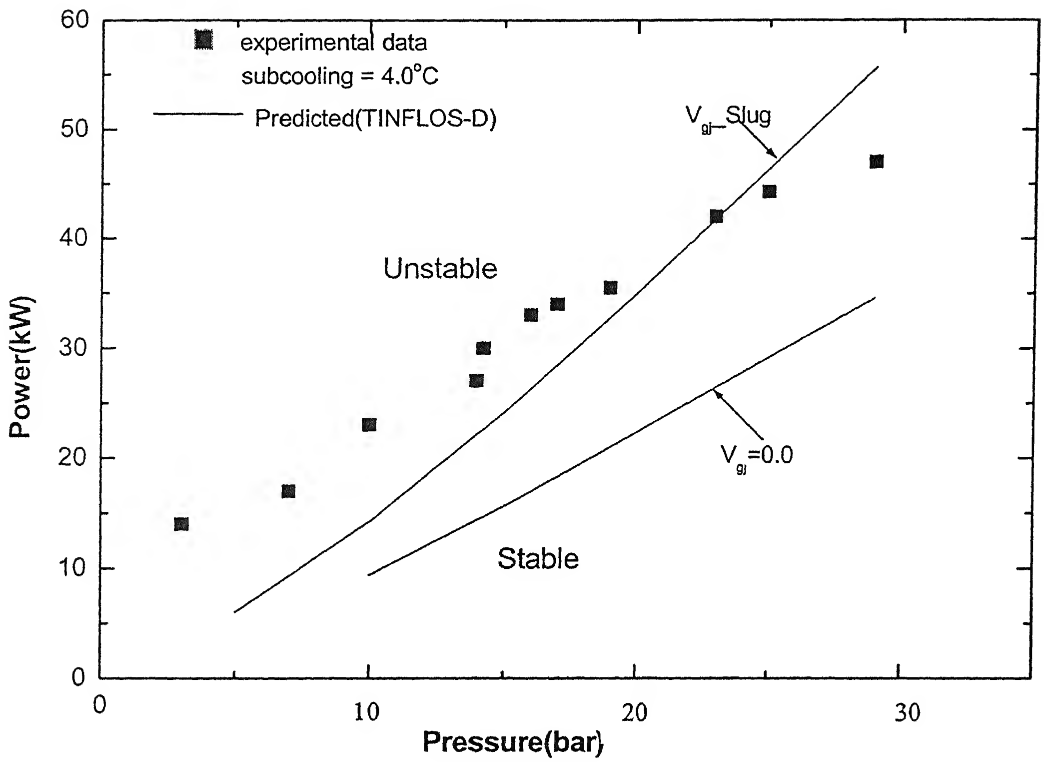


Fig 3.8 Threshold power of stability–prediction and experimental data for Furutera's loop

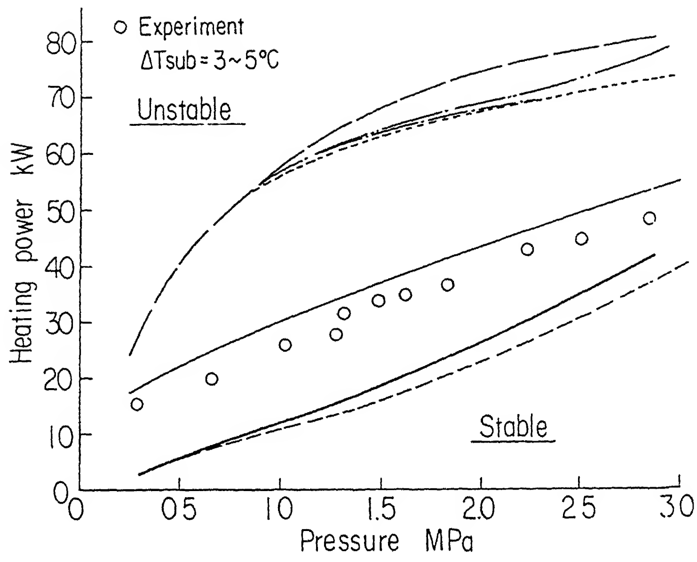


Fig. 3-5 Comparison between experimental results and theory

Table 3·6
Contents of the models

Model	Heat capacity: Single-phase liquid region	Heat capacity: Subcooled boiling region	Frictional coefficient	Curve
DST0	ignored	ignored	constant	------
DST1	considered	considered	constant	———
DST2	ignored	ignored	transient	—·—·—
DST3	considered	considered	transient	— — —
DST4	considered	ignored	transient	···········
DST5	considered	considered (Jens-Lottes)	transient	—··—
DST6	considered	ignored	constant	———

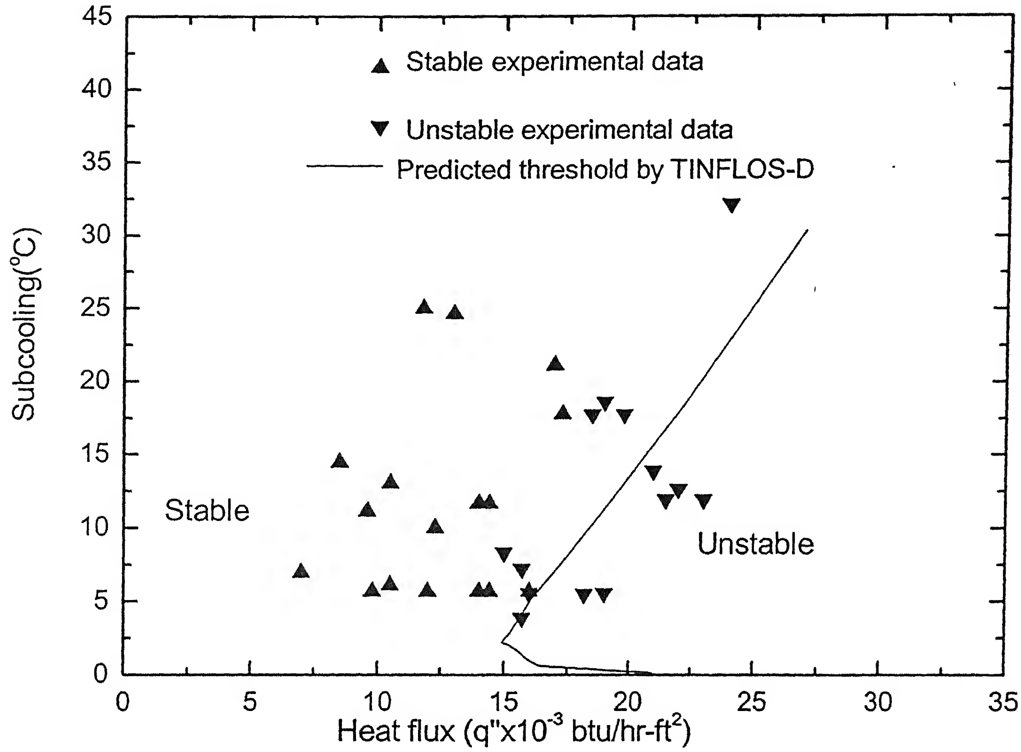


Fig 3.10 Threshold power of stability–prediction and experimental data for Chexal and Bergles loop

Table 3.7 Comparison of TINFLOS-D prediction with experimental observation for HPNCL

	Experimental Conditions / Observations				Prediction	Remarks
	Pressure	Power (kW)	Subcooling	Stable/ Unstable	Stable/ Unstable	
1	1 atm.	20.1	3.55 °C	Unstable	Unstable	Tallies with the experiment
2	2.67 bar	22.2	2.73 °C	Unstable	Unstable	Tallies with the experiment
3	1.65 bar	20.0	3.22 °C	Unstable	Unstable	Tallies with the experiment
4	5.0 bar	20.0	2.7 °C	Unstable	Unstable	Tallies with the experiment
5	5.33 bar	24.0	2.57 °C	N.S.	Unstable	Stable above 27.5 KW
6	8.06 bar	19.5	3.2 °C	Unstable	Unstable	Tallies with the experiment
7	1.18 bar	13.8	2.69 °C	Unstable	Unstable	Tallies with the experiment
8	1.39 bar	20.0	3.57 °C	Unstable	Unstable	Tallies with the experiment
9	5.86 bar	25.2	2.47 °C	Stable	Stable.	Tallies with the experiment
10	3.03 bar	31.6	2.35 °C	Stable	Unstable	Stable above 34 KW

N.S.--- NEUTRALLY STABLE POINT

Table 3.8 Comparison of TINFLOS-D predictions with experimental observation for Apsara loop

Sl. No.	Power (kW)	SubCooling(°C)	Pressure (bar)	Experimental Observation	Prediction
1	1.2644	7.60	1.504	Unstable	Unstable
2	1.33736	4.00	1.420	Unstable	Unstable
3	1.3547	4.00	1.700	Unstable	Unstable
4	1.4917	5.00	1.980	Unstable	Unstable
5	1.5886	8.12	3.300	Unstable	Unstable
6	1.6225	7.32	4.40	Unstable	Unstable
7	1.7356	7.00	4.760	Unstable	Unstable
8	1.7576	4.28	3.700	Unstable	Unstable
9	1.8695	1.90	4.500	Unstable	Unstable

CHAPTER 4

SENSITIVITY ANALYSIS OF HPNCL

4.1 Prediction of the stability behavior of the loop

The effect of various geometric and operating conditions on the flow stability behavior of the High Pressure Natural Circulation Loop (HPNCL) have been investigated by the computer code TINFLOS-D.The geometry of the HPNCL has been shown in Fig 3.1.In this HPNCL loop thermodynamic quality of the mixture is very low and is less then 2 percent. It is interesting to analyse the stability behavior of such a low quality natural circulation loop. The sensitivity analysis of HPNCL has been done for lower threshold of instability. Upper threshold occurs at higher quality. Since present analysis has been done for low quality HPNCL, our results are true for lower threshold only.

From Fig. 3.5 it can be seen that natural circulation flow rate depends on the system pressure. At low pressure, there is increase in flow rate with increase in system pressure. The initial increase in flow rate with increase in pressure is due to reduction of two-phase friction pressure drop because the two-phase friction multiplier reduces with increase in system pressure. At higher pressure, the void fraction reduces with increase in pressure and hence the two-phase mixture density increases with increase in pressure. This results in reduction of driving head at higher pressure causing the flow rate to reduce.

Density wave instability occurs normally due to the dominance of two-phase pressure drop over single-phase drop of the system. This is due to the fact that two phase pressure drop is out of phase with the change of inlet flow, while single phase pressure drop is in phase with the change of inlet flow. From the figures it can be observed that with the increase in power flow stablises. This can be explained from the fact that in low quality conditions gravitational pressure drop in two phase is dominant since frictional loss is given by:

$$\left(\Delta p_{f_{ss}} = \frac{f_m {w_{ss}}^2 L}{2D\rho_{mss}\mathrm{A}^2}\right)$$

So it is clear that it is inversely proportional to the density of the mixture, it will have small value and will not effect the stability characteristic. With increase in power since the quality increases, hence the void fraction also increases, since quality is given by the expression:

$$x = \frac{1}{1+\left(\frac{1}{\alpha}-1\right)\left(\frac{\rho_f}{\rho_g}\right)\left(\frac{V_f}{V_g}\right)}$$

The gravitational loss reduces due to decrease in two-phase mixture density. Hence the flow stablises with increase in power. It can also be observed from the figures that with increase in pressure threshold power reduces. This can be explained from the fact that with increase in pressure flow rate reduces . Outlet enthalpy is given by

$$h_{out,ss} = h_{inss} + \frac{q_w{}'' P_h}{A_C w_{ss}} - \left(\frac{\rho_f - \rho_{mss,out}}{\rho_{mss,out}} \frac{\rho_f \rho_g}{\Delta\rho} V_{gj} h_{fg}\right)$$

Where $h_{in\,ss}$ and $h_{out\,ss}$ are inlet and outlet enthalpy to the core. Due to decrease in flow rate enthalpy increases. Since the quality is given by:

$$x = \frac{h_{out} - h_f}{h_{fg}}$$

So quality also increases. Hence gravitational loss reduces due to decrease in two-phase mixture density, which accounts for the reduction in threshold power.

4.2 Effect of riser height

From Fig 4.1 it can be observed that with the increase of riser height flow stability decreases. It can be explained as follows.

With the increase of riser height downcomer height also increases. So it increases the driving force, which results in increased flow rate. Due to increase in flow rate quality decreases, as it is clear from the equation mentioned in section 4.1 for quality. This increases the gravitational pressure drop. Also the increase in riser height further increases the gravitational pressure drop.

All these effects tend to destabilize the system and the threshold power for stable flow increases.

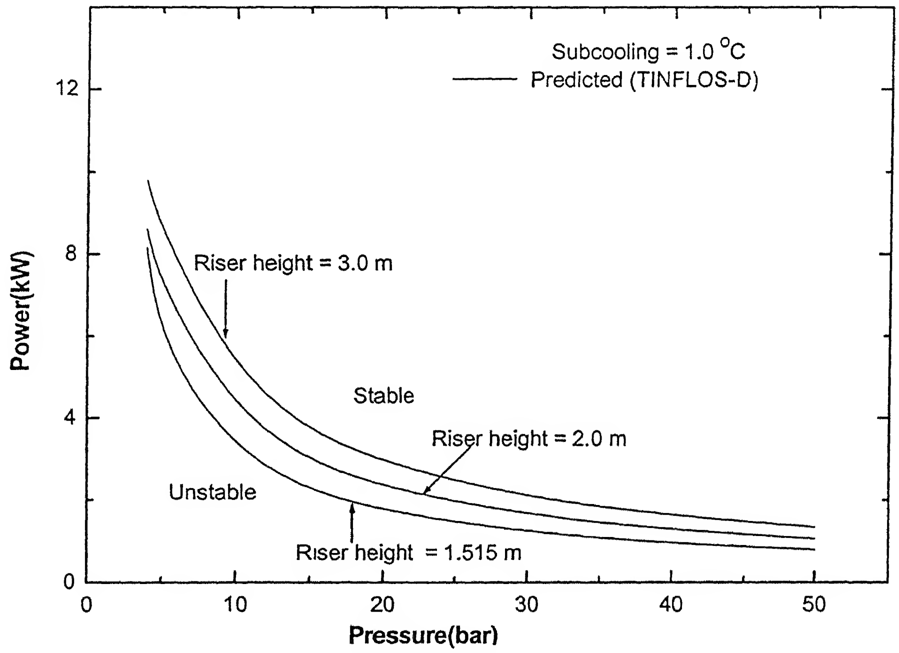


Fig 4.1 Effect of riser height on threshold of instability

4.3Effect of downcomer diameter

The effect of downcomer diameter on the flow instability is shown in Figure 4.2 .It is clear that flow destabilizes with increase in downcomer diameter. It can be explained as follows

With the increase in downcomer diameter the frictional loss reduces $\left(\Delta p_{f_{ss}} = \frac{f_m w_{ss}^{2} L}{2D\rho_{mss} A^2} \right)$, since frictional pressure drop is inversely proportional to the diameter. It results in increased flow rate. Due to increased flow rate two phase flow quality decreases (as it is clear from the equation mentioned in section 4.1 for quality). It increases the gravitational pressure drop. Hence it destabilizes the flow.

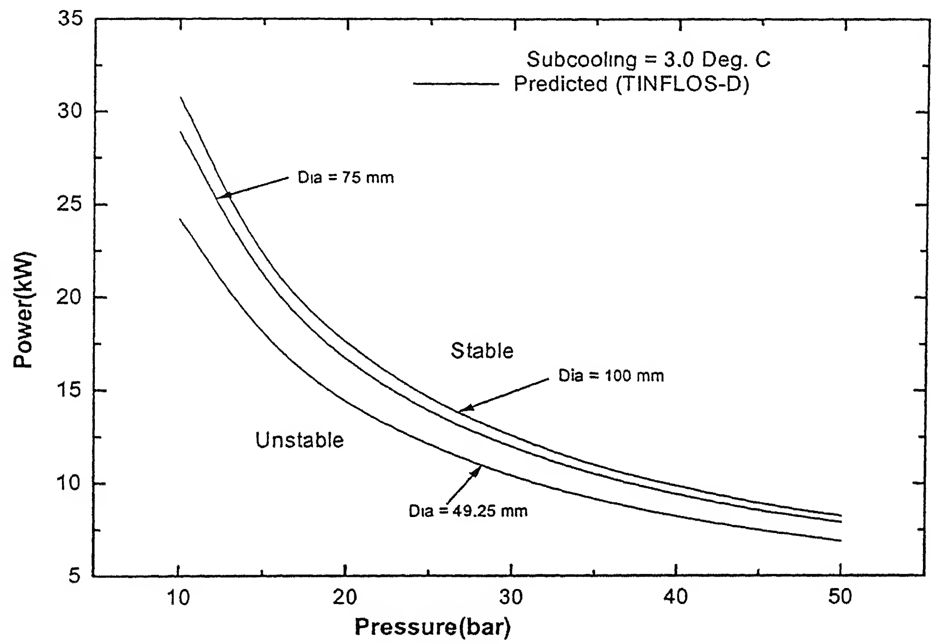


Fig 4.2 Effect of downcomer diameter on threshold of instability

4.4 Effect of core inlet loss coefficient

It is clear from the Fig 4.4 that flow stabilizes with increase in inlet loss coefficient. Due to increase in inlet restriction the pressure drop increases $\left(\because \Delta p_{lss} = \frac{K_l w_{ss}}{2A^2 \rho_f}^2\right)$. So the flow rate decreases. Hence the quality increases. So the gravitational loss reduces due to decrease in two-phase mixture density. So it stabilizes the flow.

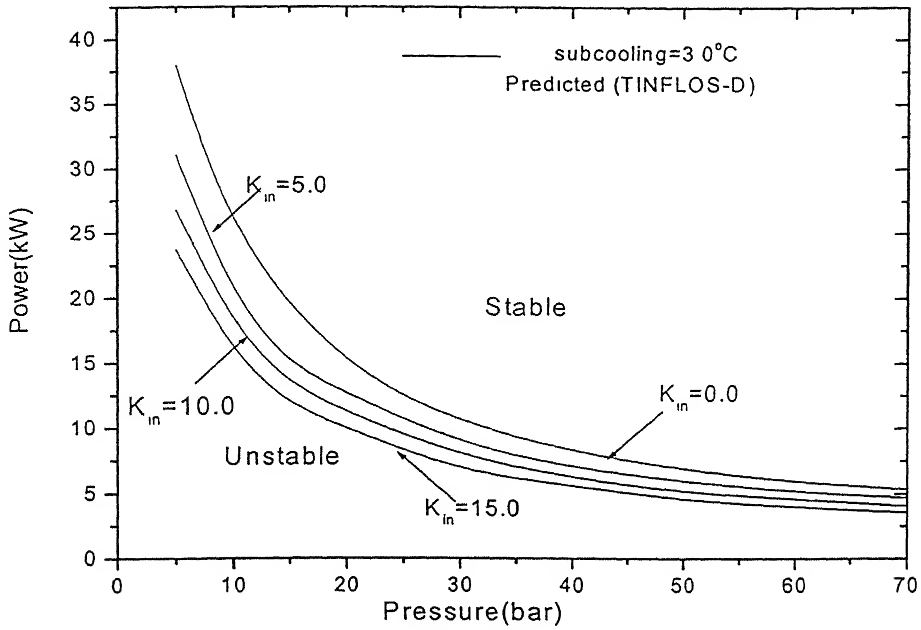


Fig4.3 Effect of core inlet coefficient on threshold of instability

4.5 Effect of core outlet loss coefficient

It is clear from fig 4.4 that outlet restriction also stabilizes the flow. It can be seen from the Figure 4.5 that it is stabilizing little more then the inlet restriction.

It is due to the fact that in the two-phase region frictional pressure drop is more due to the less density relative to single-phase flow. Since here $\left(\Delta p_{lss} = \frac{K_l w_{ss}}{2A^2 \rho_{mss}}^2\right)$ and $\rho_{mss} < \rho_f$. So here flow is relatively more reduced .it can also be seen from Fig.4.6. So quality in this case is higher than that with inlet restriction (Fig 4.7). Hence outlet restriction is stabilizing more than inlet restriction. In general outlet orificing is found to stabilize less than the outlet orificing.

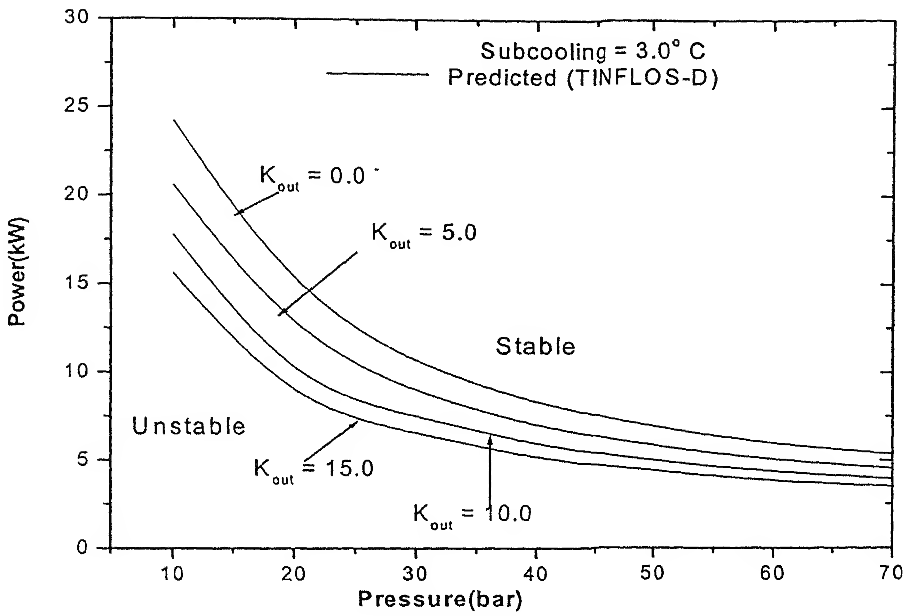


Fig4.4 Effect of core outlet coefficient on threshold of instability

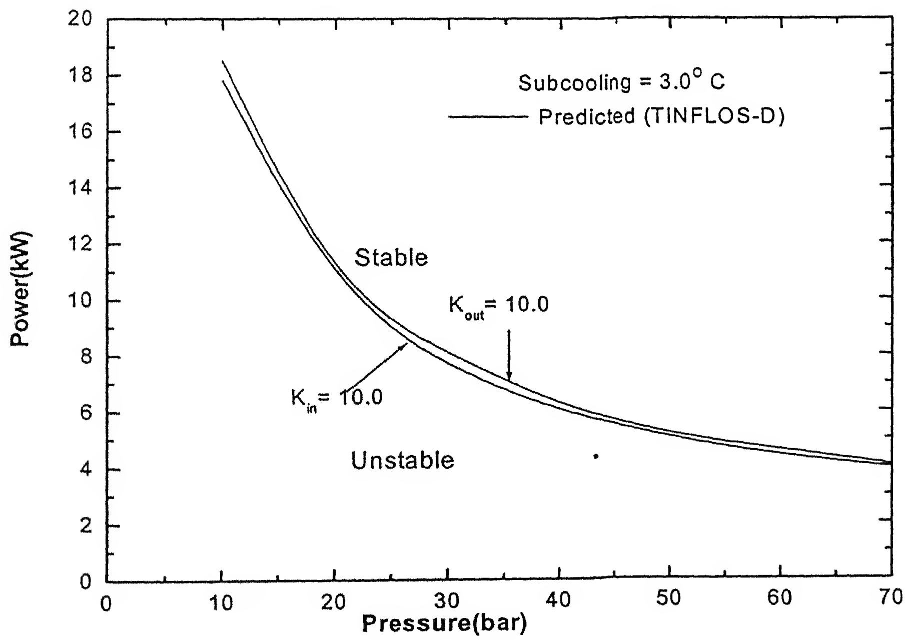


Fig 4.5 Comparison of effect of core inlet and outlet coefficient on threshold of instability

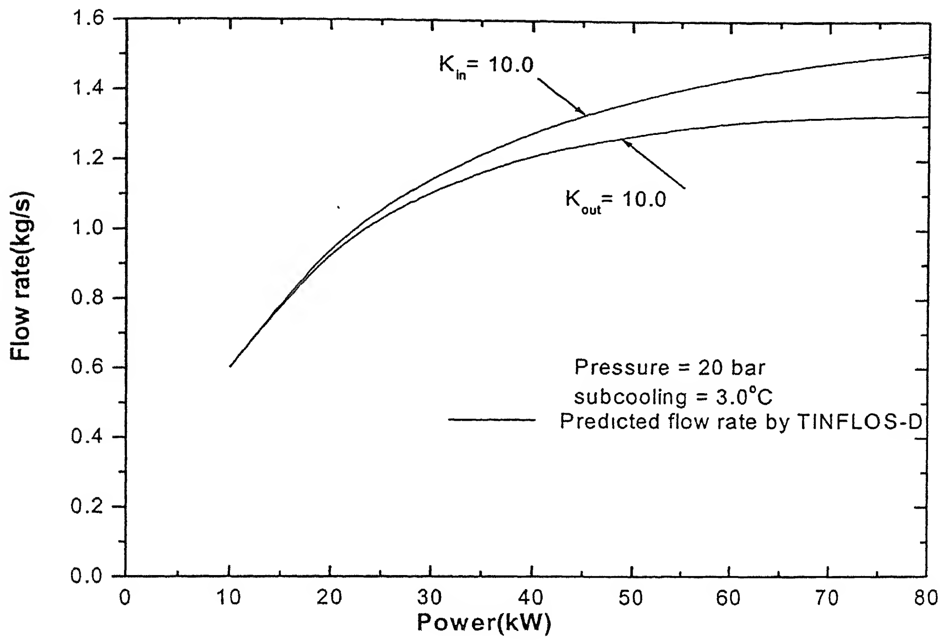


Fig 4.6 Comparison of effect of core inlet and outlet coefficient on mass flow rate

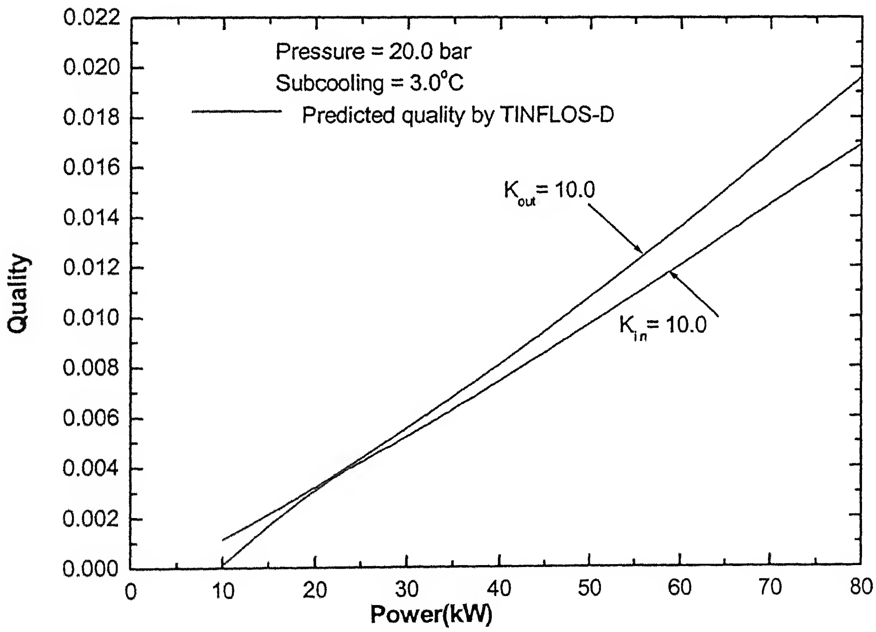


Fig 4.7 Comparison of effect of core inlet and outlet coefficient on quality

4.6 Effect of inlet subcooling

It can be seen from Figure 4.8 that with increase in inlet subcooling flow stability decreases. Due to subcooling the inlet enthalpy $h_{in,ss}$ of the fluid entering the core reduces. Hence the outlet enthalpy also decreases. So the quality is lesser (as it is clear from the equation given in section 4.1 for the quality). Hence gravitational loss increases. So, it tends to destabilize the flow.

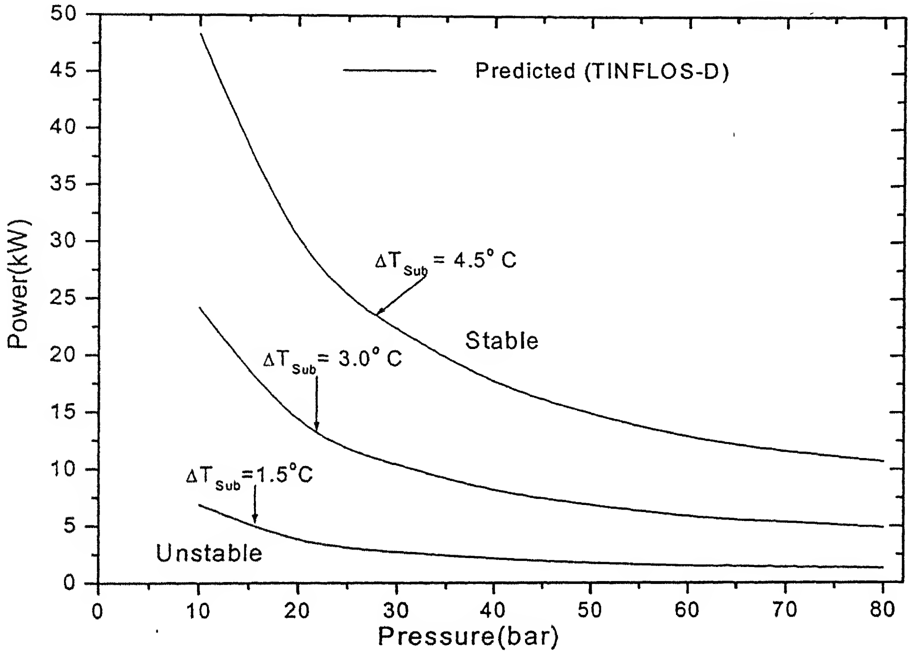


Fig4.8 Effect of intlet subcooling on threshold of instability

4.7 Effect of vapour drift velocity (V_{gj})

It can be seen from fig 4.9 that with increase in vapour drift velocity flow stabilizes. This can be explained by referring Fig 4.10.It is clear from Fig 4.10 that for the same change in quality there is less change in void fraction in the flow having higher V_{gj}. So for same change in quality there is less change in density(ρ'_m) for the flow having higher V_{gj}. So the perturbed gravitational pressure drop reduces with increase in V_{gj}. So with increase in vapour drift velocity flow stabilizes.

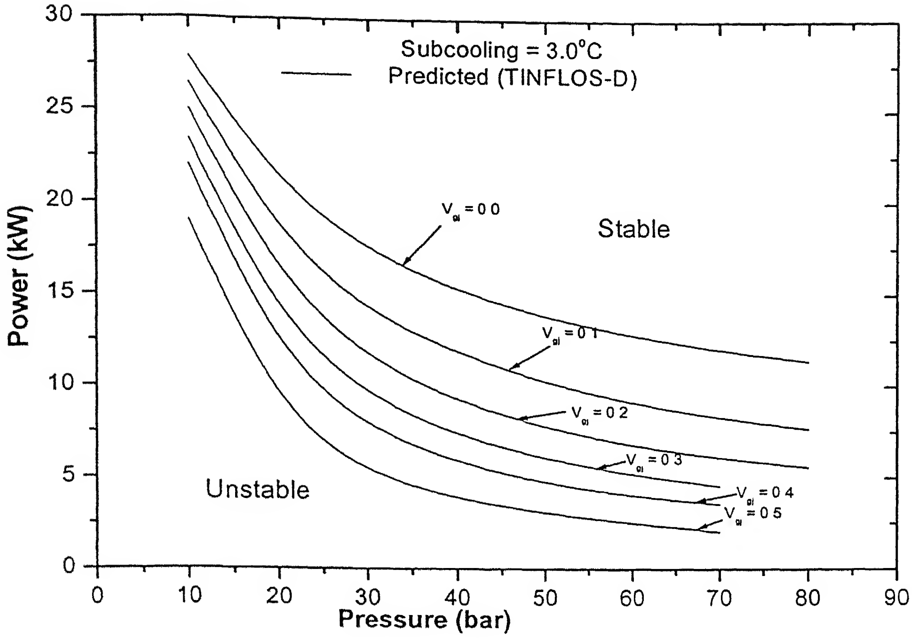


Fig4.9 Effect of vapour drift velocity on threshold of instability

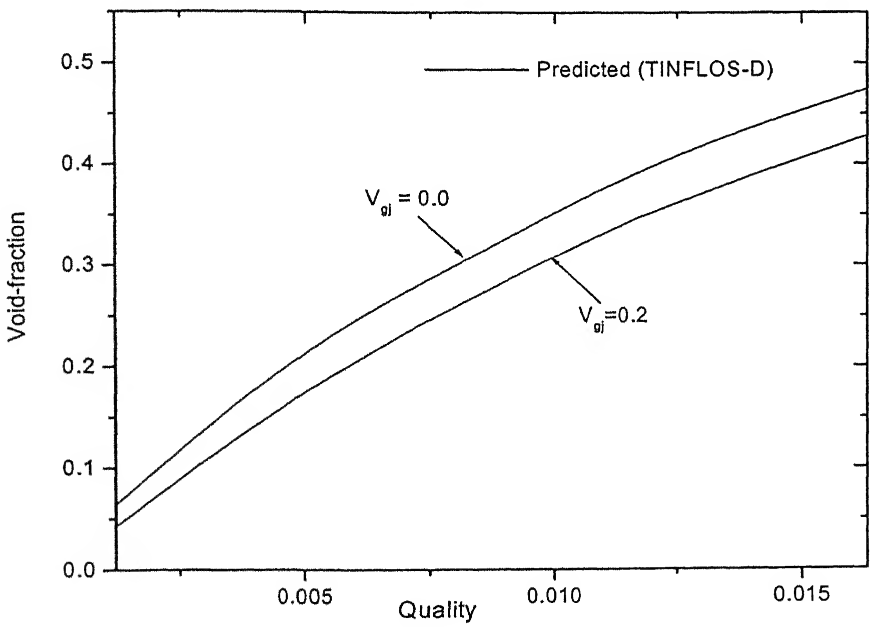


Fig 4.10 comparison of rate of change of void fraction with respect to quality

CHAPTER 5

CONCLUSIONS

The already existing computer code TINFLOS has been modified by using the drift flux model and computer code TINFLOS-D has been obtained. Model predictions were validated using the test data obtained from the four different loops (HPNCL, Apsara, Furutera's, & Bergles). Parametric sensitivity analysis has been carried out to study the flow instability behavior in HPNCL under low quality conditions (i.e. up to 2% quality).

The following insights have been obtained from this study for lower threshold of instability:

(1) The natural circulation flow rate depends on the system pressure, which initially increases then decreases with increase in pressure.

(2) With increase in system pressure system stability increases.

(3) The flow instability reduces with increase in power.

(4) Instability increases with increasing the riser height as evidenced by Fig. 4.2

(5) Instability increases with increasing the downcomer diameter as evidenced by Fig. 4.3.

(6) Increasing the resistance at the core inlet and outlet increases the stability as evidenced by Fig. 4.4 and 4.5.

(7) The effect of increase in inlet subcooling is to destablise the flow as evidenced by Fig.4.9.

(8) Flow stablises with increasing the vapour drift velocity as evidenced by Fig. 4.10.

So, it is recommended that system pressure and power should be high to enhance the stability.

5.1 Suggestions for future work

In the present code TINFLOS-D the vapour drift velocity has been assumed constant. It is taken as the drift velocity for the slug flow. While the vapour drift velocity changes with the change of flow patterns. So the vapour drift velocity based on flow pattern should be used in future to improve the code TINFLOS-D.

The vapour drift velocity for different regions is given below.

$$G = \frac{w_{ss}}{A} \qquad\qquad V_{gj} = (1-\alpha)V_r$$

(a) Bubbly regime: [35]

($G \leq 2000$ kg/$m^2 - s$ and $0.01 < \alpha \leq 0.01$)

$$V_r = \frac{1.41}{(1-\alpha)}\left[\frac{g\sigma(\rho_l - \rho_g)}{\rho_l^{\,2}}\right]^{\frac{1}{4}}$$

(b) Slug regime: [36]

($G \leq 2000$ kg/$m^2 - s$ and $0.2 < \alpha \leq 0.65$)

$$V_r = \frac{0.345}{(1-\alpha)}\left[\frac{gD_h(\rho_l - \rho_g)}{\rho_l^{\,2}}\right]^{\frac{1}{2}}$$

(c) Annular regime:[37]

($G \leq 2000$ kg/$m^2 - s$ and $0.85 < \alpha \leq 0.90$)

$$V_r = \frac{V_m}{\left(\dfrac{\rho_g(76-75\alpha)}{\rho_l\sqrt{\alpha}}\right)^{\frac{1}{2}} + \dfrac{\alpha\rho_g}{\rho_m}}$$

(d) Churn-turbulent regime:[37]

($G \geq 3000$ kg/$m^2 - s$ and for all void fractions)

$$V_r = \frac{V_m}{\dfrac{(1-C_o\alpha)}{(C_o - 1)} + \dfrac{\alpha\rho_g}{\rho_m}}$$

With Co (Void distribution parameter) =1.1 and α restricted to a maximum value of 0.8. For values of α > 0.8, V_r $(\alpha = 0.8)$ is used.

(e) For $\alpha \leq 0.005$, $\quad V_r = 0.0$

(f) For $\alpha = 1$, $\quad V_r = 0.0$

(g) Relative velocities in the transitions regions (i.e., $0.005 < \alpha \leq 0.01$, $0.1 < \alpha \leq 0.2$, $0.65 < \alpha \leq 0.85$, $0.9 < \alpha \leq 1$, for G≤ 2000 $kg/m^2 - s$, and for 2000<G<3000 $kg/m^2 - s$) are linearly interpolated in void fraction and/or mass velocity, G.

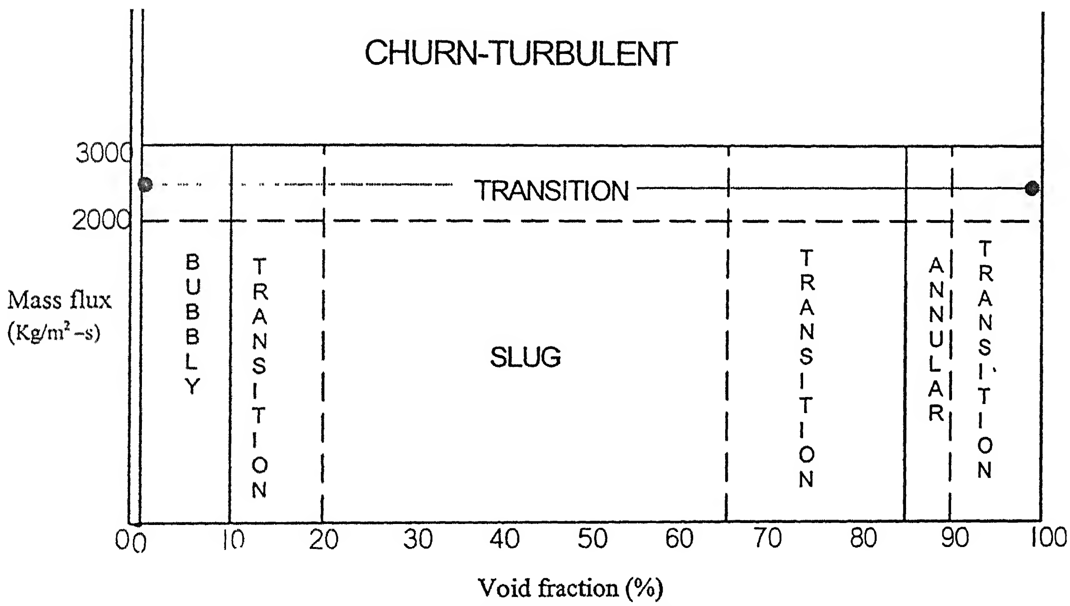


Fig 5.1 Flow regime map for slip flow correlations

Sensitivity analysis has been done for lower threshold of instability, which occurs at low quality. On increasing the power at a particular pressure, upper threshold of instability may be obtained at high qualities (around 15% quality). So in future HPNCL should be investigated for upper threshold of instability also.

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Appendix A

Identities and definitions with related derivations

α = local volumetric concentration of mixture (Void fraction)

$$\alpha = \frac{\rho_f - \rho_m}{\rho_f - \rho_g} \tag{A.1}$$

from $$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f \tag{A.2}$$

Let V_f, V_g = local velocities of liquid and vapor respectively

The volumetric flux density of the liquid is

$$j_f = (1-\alpha)\, V_f \tag{A.3}$$

and the volumetric flux density of vapor is

$$j_g = \alpha V_g \tag{A.4}$$

The volumetric flux density of the mixture is

$$j = j_f + j_g = (1-\alpha)\, V_f + \alpha V_g \tag{A.5}$$

Defining the mass velocity of the centre of mass of mixture as

$$V_m = \frac{\alpha\rho_g V_g + (1-\alpha)\rho_f V_f}{\rho_m} \tag{A.6}$$

From equation (A.5)

$$j = \alpha V_g + (1-\alpha)V_f$$

$$= V_f + \alpha(V_g - V_f)$$

$$= [\rho_m V_f + \rho_m \alpha(V_g - V_f)] / \rho_m$$

$$j = \frac{\alpha\rho_g V_f + (1-\alpha)\rho_f V_f + \alpha[\alpha\rho_g + (1-\alpha)\rho_f](V_g - V_f)}{\rho_m}$$

$$= \frac{\alpha\rho_g V_g + (1-\alpha)\rho_f V_f}{\rho_m} + \frac{\left\{ \begin{array}{l} -\alpha\rho_g V_g + \alpha^2\rho_g V_g - \alpha^2\rho_g V_f \\ +\alpha(1-\alpha)\rho_f V_g - \alpha(1-\alpha)\rho_f V_f + \alpha\rho_g V_f \end{array} \right\}}{\rho_m}$$

$$= V_m + \frac{\alpha(V_g - V_f)[(\rho_f - \rho_g) - \alpha(\rho_f - \rho_g)]}{\rho_m}$$

$$j = V_m + \frac{\alpha(\rho_f - \rho_g)(1-\alpha)V_r}{\rho_m} \qquad \text{(A.7)}$$

Where $V_r = V_g - V_f$ (A.8)

$$V_{gj} = (1-\alpha)\, V_r \qquad \text{(A.9)}$$

$$j = V_m + \frac{\alpha\Delta\rho}{\rho_m} V_{gj} \qquad \text{(A.10)}$$

$$V_m = j - \frac{\alpha\Delta\rho}{\rho_m} V_{gj} \qquad \text{(A.11)}$$

The kinematic wave velocity is defined as

$$C_k = j + V_{gj} + \alpha \frac{\partial V_{gj}}{\partial \alpha} \qquad \text{(A.12)}$$

However, if V_{gj} does not depend on α,

Then $\frac{\partial V_{gj}}{\partial \alpha} = 0.0$

In slug flow $V_{gj} = 0.345\left[\frac{gD_h(\rho_f - \rho_g)}{\rho_f{}^2}\right]^{\frac{1}{4}}$ (see reference [35]) (A.13)

It is clear from equation (A.13) that V_{gj} does not depend on α for slug flow

So $\frac{\partial V_{gj}}{\partial \alpha} = 0.0$

so $$C_k = j + V_{gj} \tag{A.14}$$

$$j = C_k - V_{gj} \tag{A.15}$$

$$C_k = j + V_g - j$$

or $$C_k = V_g = \text{local gas velocity} \tag{A.16}$$

Substituting equation (A.15) into equation (A.11), we get

$$V_m = C_k - V_{gj}\left(1 + \frac{\alpha\Delta\rho}{\rho_m}\right)$$

$$= C_k - V_{gj}\left(\frac{\alpha\rho_g + (1-\alpha)\rho_f + \alpha(\rho_f - \rho_g)}{\rho_m}\right)$$

or $$V_m = C_k - \frac{\rho_f V_{gj}}{\rho_m} \tag{A.17}$$

$$h_m = \frac{\alpha\rho_g h_g + (1-\alpha)\rho_f h_f}{\rho_m} \tag{A.18}$$

Appendix B

B.1 Single phase perturbed enthalpy equation

The single phase energy equation is given by (see equation 3,chapter 2)

$$\frac{\partial h_f}{\partial t} + V_f \frac{\partial h_f}{\partial z} = \frac{q''_w P_h}{\rho_f A_c} \tag{B.1}$$

On perturbing equation (B.1)and neglecting second order terms we get

$$\frac{\partial}{\partial t}(h_{fss} + h'_f) + (V_{fss} + V'_f)\frac{\partial}{\partial z}(h_{fss} + h'_f) = \frac{q''_w P_h}{\rho_f A_c}$$

or $$sh'_f + V_{fss}\frac{h_{fss}}{\partial z} + V_{fss}\frac{\partial h'_f}{\partial z} + V'_f \frac{\partial h_{fss}}{\partial z} = \frac{q''_w P_h}{\rho_f A_c} \tag{B.2}$$

But at steady state from equation (B.1)

$$V_{fss}\frac{\partial h_{fss}}{\partial z} = \frac{q''_w P_h}{\rho_f A_c} \tag{B.3}$$

so on putting this value in equation (B.2) we get

$$sh'_f + V_{fss}\frac{\partial h'_f}{\partial z} + V'_f \frac{q''_w P_h}{\rho_f A_c V_{fss}} = 0 \tag{B.4}$$

or $$\frac{\partial h'_f}{\partial z} + \frac{s}{V_{fss}} h'_f = -\frac{q''_w P_h}{\rho_f A_c}\frac{V'_f}{V^2_{fss}} \tag{B.5}$$

On integration of equation (B.5), we get

$$h'_f = e^{-\int \frac{s}{V_{fss}} dz}\left[\int \frac{-q''_w P_h}{V^2_{fss}\rho_f A_c} V'_f e^{\int \frac{s}{V_{fss}} dz} dz + A\right] \tag{B.6}$$

$$= e^{-\frac{sz}{V_{fss}}}\left[\frac{-q''_w P_h V'_f}{\rho_f A_c V_{fss} s} e^{\frac{sz}{V_{fss}}} + A\right] \tag{B.7}$$

Let λ_{ss} = steady state boiling boundary position since it is assumed that in single-phase region $h'_f = 0$

So putting at $z = 0$, $h'_f = 0$ in equation (B.7), we get

$$0 = \left[\frac{-q''_w P_h V'_f}{\rho_f A_c V_{fss} s} + A\right] \qquad \text{(B.8)}$$

So $$A = \frac{q''_w P_h V'_f}{\rho_f A_c V_{fss} s} \qquad \text{(B.9)}$$

so $$h'_{f\,at\,z=\lambda_{ss}} = e^{-\frac{s\lambda_{sss}}{V_{fss}}}\left[\frac{-q''_w P_h V'_f}{\rho_f A_c V_{fss} s} e^{\frac{s\lambda_{sss}}{V_{fss}}} + \frac{q''_w P_h V'_f}{\rho_f A_c V_{fss} s}\right]$$

or $$h'_{f,\lambda ss} = \frac{-q''_w P_h V'_f}{\rho_f A_c V_{fss} s}\left[1 - e^{\frac{s\lambda_{sss}}{V_{fss}}}\right] \qquad \text{(B.10)}$$

B.2 Volumetric flux propagation equation

For two phase region conservation of vapour phase

$$\frac{\partial(\alpha\rho_g)}{\partial t} + \frac{\partial}{\partial z}\left(\alpha\rho_g V_m\right) = \Gamma_g - \frac{\partial}{\partial z}\left(\frac{\alpha\rho_f\rho_g}{\rho_m}V_{gj}\right) \text{ for heated region}$$

(B.11)

Substituting equation (A.11) into equation (B.11) we get

$$\frac{\partial}{\partial t}\left(\alpha\rho_g\right) + \frac{\partial}{\partial z}\left(\alpha\rho_g J\right) = \Gamma_g - \frac{\partial}{\partial z}\left(\frac{\alpha\rho_f\rho_g V_{gj}}{\rho_m}\right) + \frac{\partial}{\partial z}\left(\frac{\alpha\rho_g\alpha V_{gj}\Delta\rho}{\rho_m}\right) \qquad \text{(B.12)}$$

$$= \Gamma_g + \frac{\partial}{\partial z}\left[\frac{\alpha\rho_g V_{gj}\{\alpha(\rho_f - \rho_g) - \rho_f\}}{\rho_m}\right]$$

$$= \Gamma_g - \frac{\partial}{\partial z}\left(\alpha\rho_g V_{gj}\right) \qquad \text{(B.13)}$$

Since $\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$

or $\rho_m = \rho_f - \alpha(\rho_f - \rho_g)$

The elementary form of the mass conservation equation is obtained from equation (B.13) as

$$\frac{\partial}{\partial t}(\alpha\rho_g) + \frac{\partial}{\partial z}(\alpha\rho_g V_g) = \Gamma_g \tag{B.14}$$

or $$\rho_g\frac{\partial\alpha}{\partial t} + \alpha\frac{\partial\rho_g}{\partial t} + \rho_g\frac{\partial}{\partial z}(\alpha V_g) + \alpha V_g\frac{\partial\rho_g}{\partial z} = \Gamma_g$$

or $$\rho_g\left[\frac{\partial\alpha}{\partial t} + \frac{\partial}{\partial z}(\alpha V_g)\right] = \Gamma_g - \alpha\frac{\partial\rho_g}{\partial t} - \alpha V_g\frac{\partial\rho_g}{\partial z}$$

or $$\frac{\partial\alpha}{\partial t} + \frac{\partial}{\partial z}(\alpha V_g) = \frac{\Gamma_g}{\rho_g} - \frac{\alpha}{\rho_g}\left[\frac{\partial\rho_g}{\partial t} + V_g\frac{\partial\rho_g}{\partial z}\right]$$

or $$\frac{\partial\alpha}{\partial t} + \frac{\partial}{\partial z}(\alpha V_g) = \frac{\Gamma_g}{\rho_g} - \frac{\alpha}{\rho_g}\frac{D\rho_g}{Dt} \tag{B.15}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_g\frac{\partial}{\partial z}$$

Similarly the elementary form of mass conservation equation for the liquid phase is

$$\frac{\partial}{\partial t}(1-\alpha)\rho_f + \frac{\partial}{\partial z}\{(1-\alpha)\rho_f V_f\} = -\Gamma_g \tag{B.16}$$

or $$\frac{\partial\rho_f}{\partial t} - \frac{\partial}{\partial t}(\alpha\rho_f) + \rho_f\frac{\partial}{\partial z}(1-\alpha)V_f + (1-\alpha)V_f\frac{\partial\rho_f}{\partial z} = -\Gamma_g$$

or $$\frac{\partial\rho_f}{\partial t} - \alpha\frac{\partial\rho_f}{\partial t} - \rho_f\frac{\partial\alpha}{\partial t} + \rho_f\frac{\partial}{\partial z}(1-\alpha)V_f + (1-\alpha)V_f\frac{\partial\rho_f}{\partial z} = -\Gamma_g$$

or $$-\rho_f\left[\frac{\partial\alpha}{\partial t}-\frac{\partial}{\partial z}(1-\alpha)V_f\right]=-\Gamma_g-(1-\alpha)\frac{\partial\rho_f}{\partial t}-(1-\alpha)V_f\frac{\partial\rho_f}{\partial z}$$

or $$-\frac{\partial\alpha}{\partial t}+\frac{\partial}{\partial z}(1-\alpha)V_f=\frac{-\Gamma_g}{\rho_f}-\frac{(1-\alpha)}{\rho_f}\left[\frac{\partial\rho_f}{\partial t}+V_f\frac{\partial\rho_f}{\partial z}\right]$$

or $$-\frac{\partial\alpha}{\partial t}+\frac{\partial}{\partial z}(1-\alpha)V_f=\frac{-\Gamma_g}{\rho_f}-\frac{(1-\alpha)}{\rho_f}\frac{D\rho_f}{Dt} \qquad \text{(B.17)}$$

Adding equations (B.17) and (B.15) together, we get

$$\frac{\partial}{\partial z}\left(\alpha V_g+(1-\alpha)V_f\right)=\Gamma_g\left(\frac{1}{\rho_g}-\frac{1}{\rho_f}\right)-\left[\frac{\alpha}{\rho_g}\frac{D\rho_f}{Dt}+\frac{(1-\alpha)}{\rho_f}\frac{D\rho_f}{Dt}\right] \qquad \text{(B.18)}$$

or $$\frac{\partial j}{\partial z}=\frac{\Gamma_g\Delta\rho}{\rho_f\rho_g}-\left[\frac{\alpha}{\rho_g}\frac{D\rho_g}{Dt}+\frac{(1-\alpha)}{\rho_f}\frac{D\rho_f}{Dt}\right] \qquad \text{(B.19)}$$

The first term on the right hand side of equation (B.19) expresses the volumetric source due to the phase change and the second is the volumetric sink due to compressibility.

it should be noted here that if each phase undergoes isochoric process the compressibility effect drops.

So, we get *Volumetric flux propagation equation*

$$\frac{\partial j}{\partial z}=\frac{\Gamma_g\Delta\rho}{\rho_f\rho_g} \qquad \text{(B.20)}$$

B.3 Density Propagation Equation

equation (B.14) can be further simplified as

$$\frac{\partial}{\partial t}\left(\alpha\rho_g\right)+\frac{\partial}{\partial z}\left(\alpha\rho_g V_g\right)=\Gamma_g \qquad \text{(B.21)}$$

or $$\rho_g \frac{\partial \alpha}{\partial t} + \alpha \frac{\partial \rho_g}{\partial t} + \rho_g \frac{\partial}{\partial z}(\alpha V_g) + V_g \alpha \frac{\partial \rho_g}{\partial z} = \Gamma_g \qquad \text{(B.22)}$$

In NCL system pressure drop is very less in comparison to operating pressure. So variation of ρ_g can be neglected (isochoric process), so

$$\rho_g \frac{\partial \alpha}{\partial t} + \rho_g \frac{\partial}{\partial z}(\alpha V_g) = \Gamma_g \qquad \text{(B.23)}$$

or $$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial z}(\alpha V_g) = \frac{\Gamma_g}{\rho_g}$$

from (A.15) and (A.16) $V_g = V_{gj} + j$

so $$\frac{\partial \alpha}{\partial t} + \frac{\partial}{\partial z}\left[\alpha(V_{gj} + j)\right] = \frac{\Gamma_g}{\rho_g}$$

or $$\frac{\partial \alpha}{\partial t} + J \frac{\partial \alpha}{\partial z} + \alpha \frac{\partial J}{\partial z} + \frac{\partial}{\partial z}(\alpha V_{gj}) = \frac{\Gamma_g}{\rho_g}$$

or $$\frac{\partial \alpha}{\partial t} + j \frac{\partial \alpha}{\partial z} + \frac{\partial}{\partial z}(\alpha V_{gj}) = \frac{\Gamma_g}{\rho_g} - \alpha \frac{\partial j}{\partial z}$$

or $$\frac{\partial \alpha}{\partial t} + \left[j + V_{gj} + \alpha \frac{\partial V_{gj}}{\partial \alpha}\right] \frac{\partial \alpha}{\partial z} = \frac{\Gamma_g}{\rho_g} - \frac{\alpha \Gamma_g \Delta\rho}{\rho_f \rho_g} \qquad \text{(B.24)}$$

or $$\frac{\partial \alpha}{\partial t} + \left[j + V_{gj} + \alpha \frac{\partial V_{gj}}{\partial \alpha}\right] \frac{\partial \alpha}{\partial z} = \frac{\Gamma_g}{\rho_g}\left[1 - \frac{\alpha \Delta\rho}{\rho_f}\right]$$

since $C_k = j + V_{gj}$ (A.14)

so $$\frac{\partial \alpha}{\partial t} + C_k \frac{\partial \alpha}{\partial z} = \frac{\Gamma_g}{\rho_g} \frac{\rho_m}{\rho_f} \qquad \text{(B.25)}$$

or $$\frac{\partial}{\partial t}\left[\frac{\rho_f - \rho_m}{\Delta\rho}\right] + C_k \frac{\partial}{\partial z}\left[\frac{\rho_f - \rho_m}{\Delta\rho}\right] = \frac{\Gamma_g}{\rho_g} \frac{\rho_m}{\rho_f}$$

or $$-\frac{1}{\Delta\rho}\left[\frac{\partial\rho_m}{\partial t}+C_k\frac{\partial\rho_m}{\partial z}\right]=\frac{\Gamma_g}{\rho_g}\frac{\rho_m}{\rho_f}$$

or $$\frac{\partial\rho_m}{\partial t}+C_k\frac{\partial\rho_m}{\partial z}=-\frac{\Gamma_g\rho_m\Delta\rho}{\rho_f\rho_g} \tag{B.26}$$

On putting $$\Omega=\frac{\Gamma_g\Delta\rho}{\rho_f\rho_g} \tag{B.27}$$

We get *density propagation equation*

$$\frac{\partial\rho_m}{\partial t}+C_k\frac{\partial\rho_m}{\partial z}=-\rho_m\Omega \tag{B.28}$$

For steady state condition

$$C_{kss}\frac{\partial\rho_{mss}}{\partial z}=-\rho_{mss}\Omega_{ss} \qquad \text{for heated region} \tag{B.29}$$

For adiabatic region $\Omega_{ss}=0.0$

so equation (B.29) can be written as

$$C_{kss}\frac{\partial\rho_{mss}}{\partial z}=0 \qquad \text{for adiabatic region} \tag{B.30}$$

B.4 Perturbed Density

On perturbing equation (B.28) we get

$$\frac{\partial}{\partial t}(\rho_{mss}+\rho'_m)+(C_{kss}+C'_k)\frac{\partial}{\partial z}(\rho_{mss}+\rho'_m)=-\Omega_{ss}(\rho_{mss}+\rho'_m)$$

or $$\frac{\partial\rho'_m}{\partial t}+C_{kss}\frac{\partial\rho_{mss}}{\partial z}+C_{kss}\frac{\partial\rho'_m}{\partial z}+C'_k\frac{\partial\rho_{mss}}{\partial z}+C'_k\frac{\partial\rho'_m}{\partial z}=-\Omega_{ss}\rho_{mss}-\Omega_{ss}\rho'_m$$

or $$s\rho'_m+C_{kss}\frac{\partial\rho'_m}{\partial z}+C'_k\frac{\partial\rho_{mss}}{\partial z}=-\Omega_{ss}\rho'_m \tag{B.31}$$

On putting the value of $\frac{\partial \rho_{mss}}{\partial z}$ from equation (B.29)

or $$s\rho'_m + C_{kss}\frac{\partial \rho'_m}{\partial z} + Ck'\frac{(-\Omega_{ss}\rho_{mss})}{Ck_{ss}} = -\Omega_{ss}\rho'_m \tag{B.32}$$

or $$(s+\Omega_{ss})\rho'_m + C_{kss}\frac{\partial \rho'_m}{\partial z} = C'_k\frac{\Omega_{ss}\rho_{mss}}{Ck_{ss}} \tag{B.33}$$

or $$\frac{\partial \rho'_m}{\partial z} + \frac{s+\Omega_{ss}}{Ck_{ss}}\rho'_m = Ck'\frac{\Omega_{ss}\rho_{mss}}{Ck_{ss}^2} \tag{B.34}$$

Solution of equation (B.34) is

Mixture density perturbation equation

$$\rho'_m = e^{-\int \frac{s+\Omega_{ss}}{C_{kss}}dz}\left[\int \frac{C'_k\Omega_{ss}\rho_{mss}}{C_{kss}^2} e^{\int \frac{s+\Omega}{C_{kss}}dz} dz + B\right] \tag{B.35}$$

Now $C_k(t) = V_{fi}(t) + \Omega_{ss}(z-\lambda(t)) + V_{gj}$

since V_{fi} and V_{gj} do not change with z

$$dC_{kss} = \Omega_{ss}\, dz \tag{B.36}$$

On substituting equation (B.36) into equation (B.35), we get

$$\rho'_m = e^{-\frac{s+\Omega_{ss}}{\Omega_{ss}}\int \frac{dC_{kss}}{C_{kss}}}\left[\int \frac{C'_k\rho_{mss}\Omega_{ss}}{C_{kss}^2}\left[e^{\int \frac{s+\Omega_{ss}}{\Omega_{ss}}\frac{dC_k}{C_{kss}}}\right]dz + B\right] \tag{B.37}$$

or $$\rho'_m = e^{-\frac{s+\Omega_{ss}}{\Omega_{ss}}\ln C_{kss}}\left[\frac{C'_k\Omega_{ss}\rho_{mss}}{C_{kss}^2}\left[e^{\frac{s+\Omega_{ss}}{\Omega_{ss}}\ln C_{kss}}\right]dz + B\right]$$

or $$\rho'_m = (Ck_{ss})^{-\frac{s+\Omega_{ss}}{\Omega_{ss}}}\left[\int \frac{C'_k\Omega_{ss}\rho_{mss}}{C_{kss}^2}(Ck_{ss})^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} dz + B\right]$$

Now from equation (B.36)

$$dz = \frac{dC_{kss}}{\Omega_{ss}}$$

so
$$\rho'_m = (C_{kss})^{-\frac{s+\Omega_{ss}}{\Omega_{ss}}}\left[\int \frac{C_k \rho_{mss}}{Ck_{ss}^2}(C_{kss})^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} dC_{kss} + B\right] \tag{B.38}$$

$$C_{kss}\frac{\partial \rho_{mss}}{\partial z} = -\Omega_{ss}\rho_{mss} \tag{B.29}$$

or
$$\frac{\partial \rho_{mss}}{\partial z} = -\frac{\Omega_{ss}\rho_{mss}}{C_{kss}}$$

or
$$\frac{\partial \rho_{mss}}{\rho_{mss}} = -\frac{\Omega_{ss}}{C_{kss}}dz$$

On integration, $\ln \rho_{mss} = -\int \frac{\Omega_{ss}}{Ck_{ss}} \frac{dCk_{ss}}{\Omega_{ss}} + C$

or $\ln \rho_{mss} = -\ln C_{kss} + C$

or
$$\rho_{mss}\, C_{kss} = C = \text{constant} \tag{B.39}$$

at $z = \lambda_{ss}$, $\rho_{mss} = \rho_f$, $C_{kss} = C_{kss,\lambda ss}$, so from (B.39) we get

$\rho_{mss}\, C_{kss} = \rho_f\, C_{kss,\lambda ss}$

or
$$\frac{\rho_{mss}}{\rho_f} = \frac{C_{kss,\lambda ss}}{C_{kss}} \tag{B.40}$$

Substituting equation (B.40) into equation (B.38), we get

$$\rho'_m = (C_{kss})^{-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\int \frac{C'_k \rho_{mss}}{C_{kss}^2}(C_{kss})^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} dC_{kss} + B\right]$$

or
$$\rho'_m = (C_{kss})^{\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\int \frac{C'_k}{C_{kss}^2}\rho_f \frac{C_{kss,\lambda ss}}{Ck_{ss}}(C_{kss})^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} dC_{kss} + B\right] \tag{B.41}$$

$$\rho_m' = \left(\mathrm{C}_{\mathrm{kss}}\right)^{-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\rho_f \mathrm{C}_{\mathrm{kss},\lambda ss}\, C_k' \frac{\Omega_{ss}}{s-\Omega_{ss}}\left(\mathrm{Ck}_{\mathrm{ss}}\right)^{\frac{s-\Omega_{ss}}{\Omega_{ss}}} + B\right] \tag{B.42}$$

In equation (B.42) to find the constant B, it can be assumed that a positive perturbation in boiling boundary results in a negative perturbation in the mixture density. To a first approximation it can be written from equation (B.39) as

$$\frac{\partial \rho_{mss}}{\partial z} = -\frac{\Omega_{ss}\rho_f}{C_{ss,\lambda ss}} = -\frac{\rho_{m,\lambda ss}'}{\lambda'} \tag{B.43}$$

$$\text{so} \qquad \rho_{m,\lambda ss}' = \frac{\Omega_{ss}\rho_f \lambda'}{C_{k\,ss,\lambda ss}} = \frac{\Omega_{ss}\rho_f}{C_{kss,\lambda ss}}\Gamma_1(s)V_{fi}' \tag{B.44}$$

Substitution of equation (B.44) into equation (B.42) for the condition that at $z = \lambda_{ss}, C_{kss,z} = C_{kss,\lambda ss}$, we get

$$\rho_{m,\lambda ss}' = \frac{\Omega_{ss}\rho_f}{C_{kss,\lambda ss}}\Gamma_1(s)V_{fi}' = \left(\mathrm{C}_{\mathrm{kss}}\right)^{-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\rho_f \mathrm{C}_{\mathrm{kss},\lambda ss}\, C_k' \frac{\Omega_{ss}}{s-\Omega_{ss}}\left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{\frac{s-\Omega_{ss}}{\Omega_{ss}}} + B\right]$$

$$\text{or} \quad \rho_{m,\lambda ss}' = \frac{\Omega_{ss}\rho_f}{C_{kss,\lambda ss}}\Gamma_1(s)V_{fi}' = \left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\rho_f \mathrm{C}_{\mathrm{k}}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{\frac{s}{\Omega_{ss}}} + B\right]$$

$$B = \frac{\Omega_{ss}\rho_f\Gamma_1(s)V_{fi}'}{C_{kss,\lambda ss}}\left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} - \rho_f \mathrm{C}_{\mathrm{k}}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\left(\mathrm{Ck}_{\mathrm{ss},\lambda ss}\right)^{\frac{s}{\Omega_{ss}}} \tag{B.45}$$

$$B = \left(\mathrm{Ck}_{\mathrm{ss},\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left[\Omega_{ss}\rho_f\Gamma_1(s)V_{fi}' - \rho_f \mathrm{Ck}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\right] \tag{B.46}$$

Substituting equation (B.46) into equation (B.42), we get

$$\rho_m' = \left(\mathrm{C}_{\mathrm{kss}}\right)^{-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}}\right)}\left[\left\{\rho_f \mathrm{Ck}_{\mathrm{ss},\lambda ss}\, \mathrm{C}_{\mathrm{k}}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\left(\mathrm{C}_{\mathrm{kss}}\right)^{\frac{s-\Omega_{ss}}{\Omega_{ss}}}\right\}\right.$$
$$\left. + \left\{\left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\Omega_{ss}\rho_f\Gamma_1(s)V_{fi}' - \rho_f \mathrm{C}_{\mathrm{k}}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\right)\right\}\right]$$

$$\rho_m' = \frac{\Omega_{ss}}{s-\Omega_{ss}}\rho_f \mathrm{C}_{\mathrm{kss},\lambda ss}\, \mathrm{C}_{\mathrm{k}}'\, C_{kss}^{-2} + \left\{\left(\mathrm{C}_{\mathrm{kss},\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\Omega_{ss}\rho_f\Gamma_1(s)V_{fi}' - \rho_f \mathrm{C}_{\mathrm{k}}' \frac{\Omega_{ss}}{s-\Omega_{ss}}\right)\left(\mathrm{C}_{\mathrm{kss}}\right)^{-\frac{s+\Omega_{ss}}{\Omega_{ss}}}\right\}$$

(B.47)

$$\rho'_m = \frac{\Omega_{ss}}{s-\Omega_{ss}}\rho_f\left(\frac{C_{kss,\lambda ss}}{C_{kss}}\right)^2\frac{1}{C_{kss,\lambda ss}}C'_k + \left\{\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}}C'_k\right\}\left(\frac{\rho_f}{C_{kss,\lambda ss}}\right)\left(\frac{C_{kss,\lambda ss}}{C_{kss}}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} \quad (B.48)$$

since $\lambda' = \Gamma_1(s)V'_{fi}$

B.5 Perturbation Equation for centre of mass velocity (for core)

On perturbing both sides of equation (A.17), we get

$$V_{mss} + V'_m = C_{kss} + C'_k - \frac{\rho_f V_{gj}}{\rho_{mss} + \rho'_m}$$

$$= C_{kss} + C'_k - \frac{\rho_f V_{gj}}{\rho_{mss}\left(1 + \frac{\rho'_m}{\rho_{mss}}\right)}$$

or $$V_{mss} + V'_m = C_{kss} + C'_k - \frac{\rho_f V_{gj}}{\rho_{mss}}\left(1 - \frac{\rho'_m}{\rho_{mss}}\right) \quad (B.49)$$

Now from equation (A.17) for steady state condition

or $$V_{mss} = C_{kss} - \frac{\rho_f V_{gj}}{\rho_{mss}}$$

so $$V'_m = C'_k + \frac{\rho_f V_{gj}\rho'_m}{\rho^2_{mss}} \quad (B.50)$$

or $$V'_m = \Gamma_2(s)V'_{fi} + \frac{\rho_f V_{gj}}{\rho^2_{mss}}\rho'_m \quad (B.51)$$

B.6 Perturbation of Density Propagation equation For Riser

Density propagation equation can be obtained by putting $\Omega = 0$ in equation (B.28),we get

so $$\frac{\partial \rho_m}{\partial t} + C_k\frac{\partial \rho_m}{\partial z} = 0 \quad (B.52)$$

On perturbing both sides, we get

$$\frac{\partial}{\partial t}\left(\rho_{mss}+\rho'_m\right)+\left(C_{kss}+C'_k\right)\frac{\partial}{\partial z}\left(\rho_{mss}+\rho'_m\right)=0$$

or $$s\rho'_m + C_{kss}\frac{\partial \rho_{mss}}{\partial z} + C_{kss}\frac{\partial \rho'_m}{\partial z} + C'_k\frac{\partial \rho'_m}{\partial z} = 0 \qquad \text{(B.53)}$$

For steady state condition equation (B.52) can be written as

$$C_{kss}\frac{\partial \rho_{mss}}{\partial z} = 0$$

So equation (B.53) can be written

$$s\rho'_m + C_{kss}\frac{\partial \rho'_m}{\partial z} + C'_k\frac{\partial \rho'_m}{\partial z} = 0 \qquad \text{(B.54)}$$

on neglecting second order term $C'_k\frac{\partial \rho'_m}{\partial z}$, we get

$$s\rho'_m + C_{kss}\frac{\partial \rho'_m}{\partial z} = 0 \qquad \text{(B.55)}$$

or $$\frac{d\rho'_m}{\rho'_m} = -\frac{s}{C_{kss}}dz$$

or $$\ln \rho'_m = -\frac{sz}{C_{kss}} + C$$ [Because Ck_{ss} does not change with z]

B.C , at $z = L$, $C_{kss} = C_{kss,L}$, $\rho'_m = \rho'_{m,L}$
(end of channel)

So $$\ln \rho'_{m,L} = \frac{-sL}{C_{kss,L}} + C$$

$$C = \ln \rho'_{m,L} + \frac{sL}{C_{kss,L}} \qquad \text{(B.56)}$$

or $$\ln \rho'_m = \frac{-sz}{C_{kss,L}} + \ln \rho'_{m,L} + \frac{sL}{C_{kss,L}}$$,or $$\ln\frac{\rho'_m}{\rho'_{m,L}} = \frac{-s}{C_{kss,L}}(z-L)$$

So perturbed density for riser is given by

$$\rho_m' = \rho_{m,L}' \, e^{-\frac{s(z-L)}{C_{kss,L}}} \tag{B.57}$$

B.7 Peturbation of centre of mass of velocity in Riser

on perturbing equation (A1.17), we get

$$V_{mss} + V_m' = C_{kss} + C_k' - \frac{\rho_f V_{gj}}{\rho_{mss} + \rho_m'}$$

$$= C_{kss} + C_k' - \frac{\rho_f V_{gj}}{\rho_{mss}\left(1 + \frac{\rho_m'}{\rho_{mss}}\right)}$$

or $$V_{mss} + V_m' = C_{kss} + C_k' - \frac{\rho_f V_{gj}}{\rho_{mss}} + \frac{\rho_f V_{gj} \rho_m'}{\rho_{mss}^2} \tag{B.58}$$

Now for steady state condition equation (A.17) can be written as

$$V_{mss} = C_{kss} - \frac{\rho_f V_{gj}}{\rho_{mss}} \tag{B.59}$$

On putting this value in equation (B.58), we get

$$V_m' = C_k' + \frac{\rho_f V_{gj} \rho_m'}{\rho_{mss}^2} \tag{B.60}$$

On putting the value of ρ_m' from equation (B.57) in equation (B.60), we get

$$V_m' = C_k' + \frac{\rho_f V_{gj}}{\rho_{mss}^2} \rho_{m,L}' \, e^{-\left[\frac{s(z-L)}{C_{kss,L}}\right]} \tag{B.61}$$

B.8 Perturbation of Pressure drop components

B.8(a) For 1-ϕ unheated regions

(1) Gravitational component

$$\Delta p_g = \int_0^L \rho_f g dz \tag{B.62}$$

on perturbing equation (B.62), we get

$$\Delta p_{gss} + \Delta p'_g = \int_0^L \rho_f g dz \tag{B.63}$$

For steady state condition of equation (B.62), we get

$$\Delta p_{gss} = \int_0^L \rho_f g dz$$

so

$$\Delta p'_g = 0.0 \tag{B.64}$$

(2) Frictional component

$$\Delta p_f = \int_0^L \frac{\rho_f f_s}{2D} V_{fi}^2 dz \tag{B.65}$$

on perturbing equation (B.65), we get

$$\Delta p_{fss} + \Delta p'_f = \int_0^L \frac{\rho_f f_s}{2D} (V_{fiss} + V'_{fi})^2 dz$$

$$= \frac{\rho_f f_s}{2D} \left(V_{fiss}^2 + V_{fi}'^2 + 2V_{fiss} V'_{fi}\right) L$$

on neglecting $V_{fi}'^2$ (second order term)

$$\Delta p_{fss} + \Delta p'_f = \frac{\rho_f f_s}{2D} \left(V_{fiss}^2 + 2V_{fiss} V'_{fi}\right) L \tag{B.66}$$

Now for steady state equation (B.65) can be written as

$$\Delta p_{fss} = \int_0^L \frac{\rho_f f_s}{2D} V_{fiss}^2 dz$$

or $$\Delta p_{fss} = \frac{\rho_f f_s}{2D} V_{fiss}^2 L \qquad \text{(B.67)}$$

From equation (B.67) and (B.66), we get

$$\Delta p'_f = \frac{\rho_f f_s}{D} V_{fiss} L V'_{fi} \qquad \text{(B.68)}$$

(3) Acceleration component

$$\Delta p_a = 0.0 \qquad \text{(B.69)}$$

So on perturbing equation (B.68)

$$\Delta p_{ass} + \Delta p'_a = 0 \qquad \text{(B.70)}$$

For steady state condition of (B.69)

$$\Delta p_{ass} = 0.0 \qquad \text{(B.71)}$$

From equation (B.70) and (B.71), we get

$$\Delta p'_a = 0.0 \qquad \text{(B.72)}$$

(4) Loss component

$$\Delta p_l = k_l \rho_f V_{fi}^2 \qquad \text{(B.73)}$$

or $$\Delta p_{lss} + \Delta p'_l = k_l \rho_f \left(V_{fiss} + V'_{fi}\right)^2$$

or $$\Delta p_{lss} + \Delta p'_l = k_l \rho_f \left(V_{fiss}^2 + 2V_{fiss} V'_{fi}\right) \qquad \text{(B.74)}$$

For steady state condition of equation (B.73)

$$\Delta p_{lss} = k_l \rho_f V_{fiss} \qquad \text{(B.75)}$$

From equation (B.74) and (B.75), we get

$$\Delta p'_l = 2k_l \rho_f V_{fiss} V'_{fi} \qquad \text{(B.76)}$$

(5) Slip component

$$\Delta p_s = 0.0 \tag{B.77}$$

so in the same way as for perturbed acceleration component, we can get

$$\Delta p'_s = 0.0 \tag{B.78}$$

(6) Inertia component

$$\Delta p_i = \int_0^L \rho_f \frac{\partial V_{fi}}{\partial t} dz \tag{B.79}$$

on perturbing both sides of equation (B.79)

$$\Delta p_{iss} + \Delta p'_i = \int_0^L \rho_f \frac{\partial}{\partial t}\left(V_{fiss} + V'_{fi}\right) dz$$

For steady state condition of equation (B.79)

$$\Delta p_{iss} = \int_0^L \rho_f \frac{\partial V_{fiss}}{\partial t} dz$$

or $\qquad \Delta p_{iss} = 0$

so $\qquad 0 + \Delta p'_i = \int_0^L \rho_f s V'_{fi} dz$

or $\qquad \Delta p'_i = \rho_f s V'_{fi} L \qquad$ (B.80)

B.8 (b) Perturbed components of Δp for single phase heated region (in core region)

(1) Gravitational component

$$\Delta p_g = \int_0^\lambda \rho_f g dz \tag{B.81}$$

On perturbing equation (B.81),we get

$$\Delta p_{gss} + \Delta p'_g = \int_0^{\lambda_{ss}+\lambda'} \rho_f g dz$$

or $\qquad \Delta p_{gss} + \Delta p'_g = \int_0^{\lambda_{ss}} \rho_f g dz + \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} \rho_f g dz \qquad$ (B.82)

For steady state condition equation (B.82) takes the form

$$\Delta p_{gss} = \int_0^{\lambda_{ss}} \rho_f g dz \qquad (B.83)$$

from equation (B.82) and equation (B.83), we get

$$\Delta p'_g = \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} \rho_f g dz$$

or $$\Delta p'_g = \rho_f g \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} dz$$

or $$\Delta p'_g = \rho_f g \lambda' \qquad (B.84)$$

or $$\Delta p'_g = \rho_f g \Gamma_1(s) V'_{fi} \qquad (B.85)$$

(2) Frictional component

$$\Delta p_f = \int_0^{\lambda} \frac{\rho_f f_s}{2D} V_{fi}^2 dz \qquad (B.86)$$

or $$\Delta p_{fss} + \Delta p'_f = \int_0^{\lambda_{ss}+\lambda'} \frac{\rho_f f_s}{2D} \left(V_{fiss} + V'_{fi}\right)^2 dz$$

or $$\Delta p_{fss} + \Delta p'_f = \frac{f_s \rho_f}{2D} \left(V_{fiss} + V'_{fi}\right)^2 (\lambda_{ss} + \lambda')$$

$$= \frac{f_s \rho_f}{2D} \left(V_{fiss}^2 + 2V_{fiss} V'_{fi}\right)(\lambda_{ss} + \lambda')$$

or $$\Delta p_{fss} + \Delta p'_f = \frac{f_s \rho_f}{2D} \left(2V_{fiss} V'_{fi}\right)(\lambda_{ss} + \lambda') + \frac{f_s \rho_f}{2D} V_{fiss}^2 (\lambda_{ss} + \lambda')$$

or $$\Delta p_{fss} + \Delta p'_f = \frac{f_s \rho_f}{2D} \left(2V_{fiss} \lambda_{ss} V'_{fi}\right) + \frac{f_s \rho_f}{2D} V_{fiss}^2 \lambda_{ss} + + \frac{f_s \rho_f}{2D} V_{fiss}^2 \lambda' \qquad (B.87)$$

Now from equation (B.86)

$\Delta p_f = \frac{\rho_f f_s}{2D} V_{fi}^2 \lambda$,hence for steady state

$$\Delta p_{fss} = \frac{\rho_f f_s}{2D} V_{fiss}^2 \lambda_{ss}$$

On putting this value in equation (B.87), we get

$$\Delta p'_f = \frac{f_s \rho_f}{D} V_{fiss} \lambda_{ss} V'_{fi} + \frac{f_s \rho_f}{2D} V_{fiss}^2 \lambda' \qquad \text{(B.88)}$$

(3) Acceleration component

$$\Delta p_a = 0.0 \qquad \text{(B.89)}$$

So as obtained in case of single phase unheated region, we can get

$$\Delta p'_a = 0.0 \qquad \text{(B.90)}$$

(4) Loss component

$$\Delta p_l = k_{il} \rho_f V_{fi}^2 \qquad \text{(B.91)}$$

now in the same way as we obtained for single phase unheated region we can get

$$\Delta p'_l = 2 k_{il} \rho_f V_{fiss} V'_{fi} \qquad \text{(B.92)}$$

(5) Slip Component

$$\Delta p_s = 0.0 \qquad \text{(B.93)}$$

So on perturbing equation (B.93), we get

$$\Delta p_{s,ss} + \Delta p'_s = 0.0$$

But for steady state condition of equation (B.93)

$$\Delta p_{s,ss} = 0.0$$

so $\qquad 0 + \Delta p'_s = 0.0$

so $\qquad \Delta p'_s = 0.0 \qquad$ (B.94)

(6) Inertia Component

$\Delta p_i = 0.0$, so in the same way as we obtained for the slip component, we can get

$$\Delta p_i' = 0.0 \tag{B.95}$$

B.9 Perturbed Pressure Drop Components in two-phase region

B.9 (a) For the core region

(1) Gravitational component

$$\Delta p_g = \int_{\lambda}^{L} \rho_m g dz \tag{B.96}$$

On perturbing equation (B.96), we get

or $$\Delta p_{gss} + \Delta p_g' = \int_{\lambda_{ss}+\lambda'}^{L} (\rho_{mss} + \rho_m') g dz$$

$$\int_{\lambda_{ss}}^{L} (\rho_{mss} + \rho_m') g dz - \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} \rho_f g dz$$

or $$\Delta p_{gss} + \Delta p_g' = \int_{\lambda_{ss}}^{L} \rho_{mss} g dz + \int_{\lambda_{ss}}^{L} \rho_m' g z - \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} \rho_f g dz \tag{B.97}$$

For steady state of equation (B.96)

$$\Delta p_{gss} = \int_{\lambda_{ss}}^{L} \rho_{mss} g dz$$

On putting this value in equation (B.97) we get

$$\Delta p_g' = \int_{\lambda_{ss}}^{L} \rho_m' g dz - \int_{\lambda_{ss}}^{\lambda_{ss}+\lambda'} \rho_f g dz \tag{B.98}$$

$$\Delta p_g' = \int_{\lambda_{ss}}^{L} \rho_m' g dz - \rho_f g \lambda'$$

On putting the value of ρ_m' from equation (B.48), we get

$$\Delta p_g' = g \int_{\lambda_{ss}}^{L} \frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f C_{kss,\lambda ss} C_k' \frac{dz}{C_{kss}^2} + g \int_{\lambda_{ss}}^{L} \left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C_k' \right)$$

$$\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}} \frac{dz}{\left(C_{kss}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}}} - \rho_f g \lambda' \tag{B.99}$$

On putting $dz = \dfrac{dC_{kss}}{\Omega_{ss}}$ from equation (B.36)

$$\Delta p'_g = g\left[\frac{\Omega_{ss}}{s-\Omega_{ss}} \frac{\rho_f}{\Omega_{ss}} C_{kss,\lambda ss} C'_k - \left[\left(C_{kss}\right)^{-1}\right]_{\lambda_{ss}}^{L}\right]$$

$$+ g\left[\left(\lambda'\Omega_{ss} - \frac{\Omega_{ss} C'_k}{s-\Omega_{ss}}\right)\frac{\rho_f}{\Omega_{ss}} (Ck_{ss})^{\frac{s}{\Omega_{ss}}} \left(\frac{-\Omega_{ss}}{s}\right)\left[(Ck_{ss})^{-\frac{s}{\Omega_{ss}}}\right]_{\lambda_{ss}}^{L}\right]$$

$$\Delta p'_f = g\left[\frac{\rho_f}{s-\Omega_{ss}} Ck_{ss,\lambda ss} Ck' - \left[\left(Ck_{ss,\lambda ss}\right)^{-1} - \left(Ck_{ss,L}\right)^{-1}\right]\right]$$

$$+ g\left[\left(+\lambda' - \frac{Ck'}{s-\Omega_{ss}}\right)\rho_f (Ck_{ss})^{\frac{s}{\Omega_{ss}}} \left(\frac{-\Omega_{ss}}{s}\right)\left[\left(Ck_{ss,L}\right)^{\frac{-s}{\Omega_{ss}}} - \left(Ck_{ss,\lambda ss}\right)^{\frac{-s}{\Omega_{ss}}}\right]\right] \tag{B.100}$$

(2) Frictional component

$$\Delta p_f = \int^{L} \frac{f_m}{2D} \rho_m V_m^2 dz = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \rho_m V_m^2 dz - \frac{f\rho_f}{2D} V_{fiss}^2 \lambda' \tag{B.101}$$

On perturbing equation (B.101)

$$\Delta p_{fss} + \Delta p'_f = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \left(\rho_{mss} + \rho'_m\right)\left(V_{mss} + V'_m\right)^2 dz - \frac{f_s}{2D} \rho_f V_{fiss}^2 \lambda' \tag{B.102}$$

or $$\Delta p_{fss} + \Delta p'_f = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \left(\rho_{mss} + \rho'_m\right)\left(V_{mss}^2 + 2V_{mss} + V'_m\right) dz - \frac{f_s}{2D} \rho_f V_{fiss}^2 \lambda'$$

$$= \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \left(\rho_{mss} V_{mss}^2 + 2V_{mss}\rho_{mss} V'_m + \rho'_m V_{mss}^2\right) dz - \frac{f_s}{2D} \rho_f V_{fiss}^2 \lambda' \tag{B.103}$$

for steady state condition of equation (B.101)

$$\Delta p_{fss} = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \rho_{mss} V_{mss}^2 dz$$

on putting this value in equation (B.103)

$$\Delta p'_f = \underbrace{\int_{\lambda_{ss}}^{L} \frac{f_m}{D} V_{mss} \rho_{mss} V'_m dz}_{Ist\,term} + \underbrace{\int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \rho'_m V_{mss}^2 dz}_{IInd\,term} - \frac{f_s}{2D} \rho_f V_{fiss}^2 \lambda' \tag{B.104}$$

$$\text{Ist term} \quad \int_{\lambda_{ss}}^{L} \frac{f_m}{D} \left(\rho_{mss} V_{mss}\right) V'_m dz$$

$$\int_{\lambda_{ss}}^{L} \frac{f_m}{D} \left(\rho_f V_{fiss}\right) V'_m dz$$

$$\text{Ist term} \quad \frac{f_m}{D} \rho_f V_{fiss} \int_{\lambda_{ss}}^{L} V'_m dz \tag{B.105}$$

on putting the value of V'_m from equation (B.51)

$$\text{Ist term} = \frac{f_m}{D} \rho_f V_{fiss} \left[\int_{\lambda_{ss}}^{L} \Gamma_2(s) V'_{fi} dz + \int_{\lambda_{ss}}^{L} \frac{\rho_f V_{gj}}{\rho_{mss}^2} \rho'_m dz \right] \tag{B.106}$$

$$= \frac{f_m}{D} \rho_f V_{fiss} \Gamma_2(s) V'_{fi} \left(L - \lambda_{ss}\right)$$

$$+ \frac{f_m}{D} \rho^2{}_f V_{fiss} V_{gj} \int_{\lambda_{ss}}^{L} \frac{\rho'_m}{\rho_{mss}^2} dz \tag{B.107}$$

Substituting equation (B.40) into equation (B.107), we get

$$\text{Ist term} = \frac{f_m}{D} \rho_f V_{fiss} \Gamma_2(s) V'_{fi} \left(L - \lambda_{ss}\right) + \left[\frac{f_m}{D} V_{fiss} V_{gj} \int_{\lambda_{ss}}^{L} \frac{\rho'_m}{C_{kss,\lambda ss}^2} C_{kss}^2 dz \right] \tag{B.108}$$

or

$$\text{Ist term} = \frac{f_m}{D} \rho_f V_{fiss} \Gamma_2(s) V'_{fi} \left(L - \lambda_{ss}\right) + \left[\frac{f_m}{D} \frac{V_{fiss} V_{gj}}{C_{kss,\lambda ss}^2} \int_{\lambda_{ss}}^{L} \rho'_m C_{kss}^2 dz \right] \tag{B.109}$$

Substituting equation (B.48) into equation (B.109), we get

$$\int_{\lambda_{ss}}^{L} \frac{f_m}{D} \rho_{mss} V_{mss} V'_m dz = \frac{f_m}{D} \rho_f V_{fiss} \Gamma_2(s) V'_{fi}(L-\lambda_{ss}) + \frac{f_m}{D} \frac{V_{fiss} V_{gj}}{C^2_{kss,\lambda ss}}$$

$$\left[\int_{\lambda_{ss}}^{L} \frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f C_{kss,\lambda ss} C'_k dz + \int_{\lambda_{ss}}^{L} \left\{ +\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \right\} \frac{\rho_f}{C_{kss,\lambda ss}} \left(\frac{C_{kss,\lambda ss}}{C_{kss}} \right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} C^2_{kss} dz \right]$$

or first term $= \frac{f_m}{D} \rho_f V_{fiss} \Gamma_2(s) V'_{fi}(L-\lambda_{ss})$

$$+ \frac{f_m}{D} \frac{V_{fiss} V_{gj}}{C^2_{kss,\lambda ss}} F \qquad \text{(B.110)}$$

where

$$F = \int_{\lambda_{ss}}^{L} \rho'_m C^2_{kss} dz \qquad \text{(B.111)}$$

or

$$F = \left[\int_{\lambda_{ss}}^{L} \frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f C_{kss,\lambda ss} C'_k dz + \int_{\lambda_{ss}}^{L} \left\{ \Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \right\} \frac{\rho_f}{C_{kss,\lambda ss}} \left(\frac{C_{kss,\lambda ss}}{C_{kss}} \right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} C^2_{kss} dz \right]$$

On putting $dz = \frac{dC_{kss}}{\Omega_{ss}}$ from equation (B.36)

$$F = \frac{\rho_f \Omega_{ss}}{s-\Omega_{ss}} C_{kss,\lambda ss} Ck' \left[\int_{\lambda_{ss}}^{L} \frac{dC_{kss}}{\Omega_{ss}} + \int_{\lambda_{ss}}^{L} \left\{ \Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \right\} \rho_f (C_{kss,\lambda ss})^{\frac{s}{\Omega_{ss}}} \frac{1}{C_{kss}^{\frac{s-\Omega_{ss}}{\Omega_{ss}}}} \frac{dC_{kss}}{\Omega_{ss}} \right]$$

(B.112)

$$F = \frac{\rho_f}{s-\Omega_{ss}} C_{kss,\lambda ss} C'_k (C_{kss,L} - C_{kss,\lambda ss}) + \left[\left\{ \lambda' - \frac{C'_k}{s-\Omega_{ss}} \right\} \rho_f C_{kss,\lambda ss}^{\frac{s}{\Omega_{ss}}} \left[\frac{C_{kss}^{\frac{2\Omega_{ss}-s}{\Omega_{ss}}}}{\frac{2\Omega_{ss}-s}{\Omega_{ss}}} \right]_{\lambda ss}^{L} \right]$$

(B.113)

$$F = \frac{\rho_f}{s-\Omega_{ss}} C_{kss,\lambda ss} C'_k \left(C_{kss,L} - C_{kss,\lambda ss}\right)$$

$$+\left[\left\{\lambda' - \frac{C'_k}{s-\Omega_{ss}}\right\}\rho_f C_{kss,\lambda ss}^{\frac{s}{\Omega_{ss}}}\left[\left(C_{kss,L}\right)^{\frac{2\Omega_{ss}-s}{\Omega_{ss}}} - \left(C_{kss,\lambda ss}\right)^{\frac{2\Omega_{ss}-s}{\Omega_{ss}}}\right]\left(\frac{\Omega_{ss}}{2\Omega_{ss}-s}\right)\right]$$

(B.114)

$$\text{IInd term} = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \rho'_m V_{mss}^2 dz$$

$$\frac{f_m}{2D}\int_{\lambda_{ss}}^{L} \rho'_m V_{mss}^2 dz \qquad \text{(B.115)}$$

On substituting equation (A.17) into equation (B.115), we get

$$\text{IInd term} = \int_{\lambda_{ss}}^{L} \frac{f_m}{2D} \rho'_m \left(C_{kss} - \frac{\rho_f V_{gj}}{\rho_{mss}}\right)^2 dz$$

$$= \frac{f_m}{2D}\int_{\lambda_{ss}}^{L}\left(C_{kss} - \frac{\rho_f V_{gj} C_{kss}}{\rho_f C_{kss,\lambda ss}}\right)^2 \rho'_m dz$$

$$= \frac{f_m}{2D}\int_{\lambda_{ss}}^{L}\left(C_{kss} - \frac{C_{kss} V_{gj}}{C_{kss,\lambda ss}}\right)^2 \rho'_m dz$$

$$= \frac{f_m}{2D}\left(1 - \frac{V_{gj}}{C_{kss,\lambda ss}}\right)^2 \int_{\lambda_{ss}}^{L} C_{kss}^2 \rho'_m dz$$

$$= \frac{f_m}{2D}\left(1 - \frac{V_{gj}}{C_{kss,\lambda ss}}\right)^2 F \qquad \text{(B.116)}$$

$$\Delta p'_f = \frac{f_m}{D}\rho_f V_{fiss}\lambda'(L-\lambda_{ss}) + \frac{f_m}{D}\frac{V_{gj}}{C_{kss,\lambda ss}^2} V_{fiss} F$$

$$+\frac{f_m}{2D}\left[1 - \frac{V_{gj}}{C_{kss,\lambda ss}^2}\right]^2 F - \frac{f_s}{2D}\rho_f V_{fiss}^2 \lambda' \qquad \text{(B.117)}$$

(3) Slip component

$$\Delta p_s = \int^{L} \frac{\partial}{\partial z}\left(\frac{\rho_f - \rho_m}{\rho_m - \rho_g}\frac{\rho_f \rho_g}{\rho_m} V_{gj}^2\right) dz$$

Assuming that $\rho_m >> \rho_g$

$$\Delta p_s = \frac{\rho_f - \rho_m}{\rho_m^2} \rho_f \rho_g V_{gj}^2 \tag{B.118}$$

On perturbing equation (B.118), we get

$$\Delta p_{s,ss} + \Delta p_s' = \frac{\rho_f - \rho_{mss,L} - \rho_{m,L}'}{\rho_{mss,L}^2 + 2\rho_{mss,L}\rho_{m,L}'} \rho_f \rho_g V_{gj}^2 \tag{B.119}$$

$$= \frac{\rho_f - \rho_{mss,L} - \rho_{m,L}'}{\rho_{mss,L}^2\left(1 + \dfrac{2\rho_{m,L}'}{\rho_{mss,L}}\right)} \rho_f \rho_g V_{gj}^2$$

$$= \frac{\rho_f - \rho_{mss,L} - \rho_{m,L}'}{\rho_{mss,L}^2}\left(1 - \frac{2\rho_{m,L}'}{\rho_{mss,L}}\right) \rho_f \rho_g V_{gj}^2$$

$$= \left\{\left[\frac{\rho_f - \rho_{mss,L}}{\rho_{mss,L}^2}\right]\left(1 - \frac{2\rho_{m,L}'}{\rho_{mss,L}}\right) - \frac{\rho_{m,L}'}{\rho_{mss,L}^2}\left(1 - \frac{2\rho_{m,L}'}{\rho_{mss,L}}\right)\right\} \rho_f \rho_g V_{gj}^2$$

$$= \frac{\rho_f - \rho_{mss,L}}{\rho_{mss,L}^2} \rho_f \rho_g V_{gj}^2 - \frac{2\rho_{m,L}'(\rho_f - \rho_{mss,L})}{\rho_{mss,L}^3} \rho_f \rho_g V_{gj}^2 - \frac{\rho_{m,L}' \rho_f \rho_g V_{gj}^2}{\rho_{mss,L}^2}$$

or $$\Delta p_s' = \left\{\frac{-2\rho_{m,L}' \rho_f^2 \rho_g V_{gj}^2}{\rho_{mss,L}^3}\right\} + \frac{2\rho_{m,L}' \rho_f \rho_g V_{gj}^2}{\rho_{mss,L}^2} - \frac{\rho_{m,L}' \rho_f \rho_g V_{gj}^2}{\rho_{mss,L}^2} \tag{B.120}$$

or $$\Delta p_s' = \frac{\rho_{m,L}' \rho_f \rho_g V_{gj}^2}{\rho_{mss,L}^2} - \frac{2\rho_{m,L}' \rho_f^2 \rho_g V_{gj}^2}{\rho_{mss,L}^3}$$

or $$\Delta p_s' = \frac{\rho_{m,L}' \rho_f \rho_g V_{gj}^2}{\rho_{mss,L}^2}\left[1 - \frac{2\rho_f}{\rho_{mss,L}}\right] \tag{B.121}$$

(4) Inertia component

$$\Delta p_i = \int^{L} \rho_m \frac{\partial V_m}{\partial t} dz \tag{B.122}$$

On perturbing equation (B.122), we get

$$\Delta p_{iss} + \Delta p_i' = \int_{l_{ss}+\lambda'}^{L} (\rho_m + \rho_m') \frac{\partial (V_{mss} + V_m')}{\partial t} dz$$

$$= \int_{l_{ss}+\lambda'}^{L} (\rho_m + \rho_m') \left(\frac{\partial V_{mss}}{\partial t} + \frac{\partial V_m'}{\partial t} \right) dz$$

$$= \int_{l_{ss}+\lambda'}^{L} (\rho_{mss} + \rho_m') \frac{\partial V_m'}{\partial t} dz$$

$$\left(\text{since } \frac{\partial V_{mss}}{\partial t} = 0 \right)$$

$$= \int_{l_{ss}+\lambda'}^{L} (\rho_{mss} + \rho_m') s V_m' dz \tag{B.123}$$

On neglecting the second order term in (B.123), We get

or $$\Delta p_{iss} + \Delta p_i' = \int_{l_{ss}+\lambda'}^{L} s \rho_{mss} V_m' dz$$

or $$\Delta p_{iss} + \Delta p_i' = \int_{l_{ss}}^{L} s \rho_{mss} V_m' dz - \int_{l_{ss}}^{l_{ss}+\lambda'} s \rho_f V_{fi}' dz$$

or $$\Delta p_{iss} + \Delta p_i' = \int_{l_{ss}}^{L} s \rho_{mss} V_m' dz - s \rho_f V_{fi}' \lambda'$$

on neglecting second order term,we get

or $$\Delta p_{iss} + \Delta p_i' = \int_{l_{ss}}^{L} s \rho_{mss} V_m' dz \tag{B.124}$$

now for steady state condition of equation (B.122) can be written as

$$\Delta p_{iss} = \int_{\lambda_{ss}}^{L} \rho_{mss} \frac{\partial V_{mss}}{\partial t} dz = 0$$

so $$\Delta p'_i = \int_{\lambda_{ss}}^{L} s\rho_{mss} V'_m dz \tag{B.125}$$

on putting the value of V'_m from equation (B.51), and value of ρ_{mss} from equation (B.40), we get

$$\Delta p'_i = \int_{\lambda_{ss}}^{L} \frac{\rho_f C_{kss,\lambda ss}}{C_{kss}} s\left(\Gamma_2(s) V'_{fi} + \frac{V_{gj}}{\rho_f}\frac{C^2_{kss}}{C^2_{kss,\lambda ss}}\rho'_m\right)dz$$

or $$\Delta p'_i = \int_{\lambda_{ss}}^{L} \frac{\rho_f C_{kss,\lambda ss}}{C_{kss}} s\Gamma_2(s)V'_{fi}dz + \int_{\lambda_{ss}}^{L} V_{gj} s \frac{C_{kss}}{C_{kss,\lambda ss}}\rho'_m dz$$

now $dz = \frac{dC_{kss}}{\Omega_{ss}}$ from equation (B.36)

so $$\Delta p'_i = \rho_f C_{kss,\lambda ss} s\Gamma_2(s)V'_{fi}\int_{\lambda_{ss}}^{L} \frac{dC_{kss}}{\Omega_{ss}C_{kss}} + \frac{V_{gj}s}{C_{kss,\lambda ss}}\int_{\lambda_{ss}}^{L} Ck_{ss}\rho'_m \frac{dC_{kss}}{\Omega_{ss}}$$

$$= \frac{\rho_f C_{kss,\lambda ss} s\Gamma_2(s)V'_{fi}}{\Omega_{ss}} \ln\frac{C_{kss,L}}{C_{kss,\lambda ss}} + \frac{V_{gj}s}{\Omega_{ss}C_{kss,\lambda ss}}\int_{\lambda_{ss}}^{L} C_{kss}\rho'_m dC_{kss}$$

$$\int_{\lambda_{ss}}^{L} Ck_{ss}\rho'_m dCk_{ss} = \int_{\lambda_{ss}}^{L} \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k\rho_f \frac{C_{kss,\lambda ss}}{C_{kss}} dC_{kss}$$

$$+ \int_{\lambda_{ss}}^{L} C_{kss}\left\{\Omega_{ss}\lambda' - \frac{\Omega_{ss}C''_k}{s-\Omega_{ss}}\right\}\frac{\rho_f}{C_{kss,\lambda ss}}\left(\frac{C_{kss,\lambda ss}}{C_{kss}}\right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} dC_{kss}$$

$$= \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \rho_f C_{k\,ss,\lambda ss} \ln\left(\frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}\right)$$

$$+ \left(C_{k\,ss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}} \rho_f \left\{+\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k\right\} \int_{\lambda_{ss}}^{L} \frac{dC_{k\,ss}}{\left(C_{k\,ss}\right)^{\frac{s}{\Omega_{ss}}}}$$

$$= \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \rho_f C_{k\,ss,\lambda ss} \ln \frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}$$

$$+ \left(C_{k\,ss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}} \rho_f \left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k\right)\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)\left[\left(C_{k\,ss,L}\right)^{\frac{-s+\Omega_{ss}}{\Omega_{ss}}} - \left(C_{k\,ss,\lambda ss}\right)^{\frac{-s+\Omega_{ss}}{\Omega_{ss}}}\right]$$

so $$\Delta p'_i = \int_{\lambda ss}^{L} \rho_{mss} s V'_m dz$$

$$= \frac{\rho_f C_{k\,ss,\lambda ss} s I_2(s) V'_{fi}}{\Omega_{ss}} \ln \frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}$$

$$+ \frac{V'_{gj} s}{C_{k\,ss,\lambda ss}\Omega_{ss}} \left[\frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k \rho_f C_{k\,ss,\lambda ss} \ln \frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}\right.$$

$$\left. + \rho_f \left\{-C'_k \frac{\Omega_{ss}}{s-\Omega_{ss}} + \Omega_{ss}\lambda'\right\} \frac{\Omega_{ss}}{\Omega_{ss}-s} C_{k\,ss,\lambda ss} \left\{\left(\frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - 1.0\right\}\right]$$

or $$\Delta p'_i = \frac{\rho_f C_{k\,ss,\lambda ss} s I_2(s) V'_{fi}}{\Omega_{ss}} \ln \frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}$$

$$+ \frac{V'_{gj} s}{C_{k\,ss,\lambda ss}} \left[\frac{C'_k}{s-\Omega_{ss}} \rho_f C_{k\,ss,\lambda ss} \ln \frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}\right.$$

$$\left. + \rho_f \left\{\Omega_{ss}\lambda' - \left(\frac{C'_k \Omega_{ss}}{s-\Omega_{ss}}\right)\right\}\left(\frac{C_{k\,ss,\lambda ss}}{-s+\Omega_{ss}}\right)\left\{\left(\frac{C_{k\,ss,L}}{C_{k\,ss,\lambda ss}}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - 1.0\right\}\right] \qquad (B.126)$$

(5) Acceleration term

$$\Delta p_a = \int_{L_{ss}}^{L} \rho_m V_m \frac{\partial V_m}{\partial z} dz \tag{B.127}$$

mass conservation equation is

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m V_m) = 0$$

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial \rho_m}{\partial z} V_m + \rho_m \frac{\partial V_m}{\partial z} = 0$$

so $$\rho_m \frac{\partial V_m}{\partial z} = -\frac{\partial \rho_m}{\partial t} - \frac{\partial \rho_m}{\partial z} V_m \tag{B.128}$$

On putting this value in equation (B.127), we get

$$\Delta p_a = \int_{L_{ss}}^{L} V_m \left(\frac{-\partial \rho_m}{\partial t} - V_m \frac{\partial \rho_m}{\partial z} \right) dz$$

$$\Delta p_a = -\int_{L_{ss}}^{L} V_m \frac{\partial \rho_m}{\partial t} dz - \int_{L_{ss}}^{L} V_m^2 \frac{\partial \rho_m}{\partial z} dz \tag{B.129}$$

On perturbing both sides of equation (B.129), we get

$$\Delta p_{ass} + \Delta p_s' = -\int_{L_{ss}}^{L} (V_{mss} + V')_m \left(\frac{\partial \rho_{mss}}{\partial t} + \frac{\partial \rho_m'}{\partial z} \right) dz$$

$$- \int_{L_{ss}}^{L} (V_{mss} + V_m')^2 \frac{\partial}{\partial z} (\partial \rho_{mss} + \partial \rho_m') dz$$

$$= -\int_{L_{ss}}^{L} V_{mss} \frac{\partial \rho_m'}{\partial t} dz - \int_{L_{ss}}^{L} V_{mss}^2 \frac{\partial \rho_{mss}}{\partial z} dz - \int_{L_{ss}}^{L} V_{mss}^2 \frac{\partial \rho_m'}{\partial z} dz$$

$$- \int_{L_{ss}}^{L} 2 V_{mss} V_m' \frac{\partial \rho_{mss}}{\partial z} dz \qquad \left(\because \frac{\partial \rho_{mss}}{\partial t} = 0.0 \right) \tag{B.130}$$

For steady state condition from equation (B.129), we get

$$\Delta p_{a ss} = -\int_{ss}^{L} V_{mss}^2 \frac{\partial \rho_{mss}}{\partial z} dz \tag{B.131}$$

From equation (B.130) and (B.131), we get

$$\Delta p'_a = -\int_{ss}^{L} V_{mss} s\rho'_m dz - \int_{ss}^{L} V_{mss}^2 \frac{\partial \rho'_m}{\partial z} dz$$

$$-\int_{ss}^{L} 2V_{mss} V'_m \frac{\partial \rho_{mss}}{\partial z} dz \tag{B.132}$$

$$\int_{ss}^{L} V_{mss} s\rho'_m dz = \int_{ss}^{L} \left(C_{kss} - \frac{\rho_f V_{gj}}{\rho_{mss}} \right) s\rho'_m dz \tag{B.133}$$

[Putting equation (A.17) for V_{mss}]

Now on putting the value of ρ_{mss} from equation (B.40) into equation (B.133), we get

$$\int_{ss}^{L} V_{mss} s\rho'_m dz = \int_{ss}^{L} C_{kss} \left(1 - \frac{V_{gj}}{C_{kss,\lambda ss}} \right) s\rho'_m dz$$

$$= \left(1 - \frac{V_{gj}}{C_{kss,\lambda ss}} \right) s \int_{ss}^{L} C_{kss} \rho'_m dz$$

$$= \left(1 - \frac{V_{gj}}{C_{kss,\lambda ss}} \right) s \int_{ss}^{L} C_{kss} \left[\frac{\Omega_{ss}}{s - \Omega_{ss}} \rho_f \frac{C_{kss,\lambda ss}}{C_{kss}^2} C'_k \right.$$

$$\left. + \left\{ \Omega_{ss} \lambda' - \frac{\Omega_{ss}}{s - \Omega_{ss}} C'_k \right\} \frac{\rho_f}{Ck_{ss,\lambda ss}} \left(\frac{C_{kss,\lambda ss}}{C_{kss}} \right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} \right] dz$$

on putting value of dz from equation (B.36)

$$= \left(1 - \frac{V_{gj}}{Ck_{ss,\lambda ss}} \right) s \frac{\Omega_{ss}}{s - \Omega_{ss}} C'_k \rho_f C_{kss,\lambda ss} \int_{ss}^{L} \frac{dC_{kss}}{\Omega_{ss} C_{kss}}$$

$$+\left(1-\frac{V_{gl}}{C_{kss,\lambda ss}}\right)\frac{s}{\Omega_{ss}}\left\{\Omega_{ss}\lambda'-\frac{\Omega_{ss}}{s-\Omega_{ss}}C'_k\right\}\rho_f\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\int_{\lambda_{ss}}^{L}\frac{dC_{kss}}{\left(C_{kss}\right)^{\frac{s}{\Omega_{ss}}}}$$

$$=\left(1-\frac{V_{gl}}{C_{kss,\lambda ss}}\right)\frac{s}{s-\Omega_{ss}}C'_k\rho_f C_{kss,\lambda ss}\left[\ln\frac{C_{kss,L}}{C_{kss}\lambda_{ss}}\right]$$

$$+\left(1-\frac{V_{gl}}{C_{kss,\lambda ss}}\right)s\left(\lambda'-\frac{C'_k}{s-\Omega_{ss}}\right)\rho_f\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)$$

$$\left[\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}-\left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right]$$

$$=\left(1-\frac{V_{gl}}{C_{kss,\lambda ss}}\right)\left[\frac{s}{s-\Omega_{ss}}C'_k\rho_f C_{kss,\lambda ss}\left\{\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right\}\right.$$

$$\left.+s\left\{\lambda'-\frac{C'_k}{s-\Omega_{ss}}\right\}\rho_f\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}-\left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\right]$$

(B.134)

now $$\int_{\lambda_{ss}}^{L} V_{mss}^2\frac{\partial\rho'_m}{\partial z}dz=\int_{\lambda_{ss}}^{L}\frac{\rho_{mss}^2V_{mss}^2}{\rho_{mss}^2}\frac{\partial\rho'_m}{\partial z}dz \qquad (B.135)$$

now $\rho_{mss}V_{mss}=\rho_f V_{fiss}$ from conservation of mass and putting the value of $\frac{\partial\rho'_m}{\partial z}$ from

equation (B.34), we get

$$\int_{\lambda_{ss}}^{L} V_{mss}^2\frac{\partial\rho'_m}{\partial z}=\int_{\lambda_{ss}}^{L}\frac{\rho_f V_{fiss}^2}{\rho_{mss}^2}\left[-\left(\frac{s+\Omega_{ss}}{C_{kss}}\right)\rho'_m dz+\frac{\Omega_{ss}\rho_{mss}}{C_{kss}^2}C'_k dz\right]$$

on putting the value of ρ_{mss} from equation (B.40), we get

$$\int_{\lambda_{ss}}^{L} V_{mss}^2\frac{\partial\rho'_m}{\partial z}=\rho_f V_{fiss}^2\left[\int_{\lambda_{ss}}^{L}\left\{\frac{-\left(s+\Omega_{ss}\right)}{C_{kss,\lambda ss}^2\rho_f^2}\right\}C_{kss}\rho'_m dz+\int_{\lambda_{ss}}^{L}\frac{\Omega_{ss}C'_k}{\rho_f C_{kss,\lambda ss}}\frac{dz}{C_{kss}}\right.$$

$$= \rho_f V_{fiss}^2 \left[-\frac{(s+\Omega_{ss})}{\rho_f^2 Ck_{ss,\lambda ss}^2} \int_{\lambda_{ss}}^{L} C_{kss}\rho_m' \frac{dC_{kss}}{\Omega_{ss}} + \frac{\Omega_{ss}}{\rho_f C_{kss,\lambda ss}} \int_{\lambda_{ss}}^{L} C_k' \frac{dz}{C_{kss}} \right]$$

$$= \rho_f V_{fiss}^2 \left[-\left(\frac{s+\Omega_{ss}}{\Omega_{ss}} \right) \frac{1}{\rho_f^2 C_{kss,\lambda ss}^2} \int C_{kss}\rho_m' dC_{kss} + \frac{1}{\rho_f C_{kss,\lambda ss}} \int_{\lambda_{ss}}^{L} C_k' \frac{dC_{kss}}{C_{kss}} \right]$$

(B.136)

now $\int_{\lambda_{ss}}^{L} C_{kss}\rho_m' dC_{kss}$

$$= \int_{\lambda_{ss}}^{L} C_{kss} \left\{ \frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f \frac{C_{kss,\lambda ss}}{C_{kss}^2} C_k' + \left(\Omega_{ss}\lambda' - \frac{\Omega_{ss} C_k'}{s-\Omega_{ss}} \right) \frac{\rho_f}{C_{kss,\lambda ss}} \left(\frac{C_{kss,\lambda ss}}{C_{kss}} \right)^{\frac{s+\Omega_{ss}}{\Omega_{ss}}} \right\} dC_{kss}$$

(on putting the value of ρ_m' from eqn. (B.48))

$$= \int_{\lambda_{ss}}^{L} \frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f \frac{C_{kss,\lambda ss}}{Ck_{ss}} C_k' dC_{kss} + \int_{\lambda_{ss}}^{L} \left(\Omega_{ss}\lambda' - \frac{\Omega_{ss} C_k'}{s-\Omega_{ss}} \right) \rho_f \left(\frac{C_{kss,\lambda ss}}{C_{kss}} \right)^{\frac{s}{\Omega_{ss}}} dC_{kss}$$

$$= \left[\frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f C_{kss,\lambda ss} C_k' \ln\left(\frac{C_{kss,L}}{C_{kss,\lambda ss}} \right) \right] + \left[\left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C_k' \right) \rho_f (C_{kss,\lambda ss})^{\frac{s}{\Omega_{ss}}} \left(\frac{\Omega_{ss}}{\Omega_{ss}-s} \right) \left\{ (C_{kss,L})^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - (C_{kss,\lambda ss})^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} \right\} \right] \qquad (B.137)$$

On substituting equation (B.137) into equation (B.136), we get

$$\int_{\lambda_{ss}}^{L} V_{mss}^2 \rho_m' dz = \rho_f^2 V_{fiss}^2 \left\{ -\left(\frac{s+\Omega_{ss}}{\Omega_{ss}} \right) \right\} \frac{1}{\rho_f^2 C_{kss,\lambda ss}^2} \left[\frac{\Omega_{ss}}{s-\Omega_{ss}} \rho_f C_{kss,\lambda ss} C_k' \ln\left(\frac{C_{kss,L}}{C_{kss,\lambda ss}} \right) \right.$$

$$+\left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}}C_k'\right)\rho_f\left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)$$

$$\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - \left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\Bigg] + \frac{\rho_f^2 V_{fiss}^2}{\rho_f C_{kss,\lambda ss}}C_k' \ln\left(\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right) \qquad \text{(B.138)}$$

$$= -V_{fiss}^2\left(\frac{s+\Omega_{ss}}{C^2{}_{kss,\lambda ss}}\right)\left[\frac{1}{s-\Omega_{ss}}\rho_f C_{kss,\lambda ss}C_k' \ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right.$$

$$+\left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}}C_k'\right)\rho_f\frac{C_{k\,ss,\lambda ss}^{\frac{s}{\Omega_{ss}}}}{\Omega_{ss}-s}\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - \left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\Bigg]$$

$$+\frac{\rho_f V_{fiss}^2 Ck'}{Ck_{ss,\lambda ss}}\ln\frac{C_{kss,L}}{C_{kss,\lambda ss}} \qquad \text{(B.139)}$$

now $\int_{\lambda_{ss}}^{L} 2\mathrm{V}_{\mathrm{mss}}\mathrm{V}_{\mathrm{m}}' \frac{\partial \rho_{\mathrm{mss}}}{\partial \mathrm{z}}\mathrm{dz}$

$$= \int_{\lambda_{ss}}^{L}\frac{2\rho_f V_{fiss}}{\rho_{mss}}\left[C_k' + \frac{\rho_f V_{gj}}{\rho_{mss}^2}\rho_m'\right]\left(\frac{-\rho_{mss}\Omega_{ss}}{C_{kss}}\right)dz$$

(Putting equation (B.50) and equation (B.29))

$$= -2\rho_f V_{fiss}\Omega_{ss}\int_{\lambda_{ss}}^{L}\left(C_k' + \frac{\rho_f V_{gj}}{\rho_{mss}^2}\rho_m'\right)\frac{dz}{C_{kss}} \qquad \text{(B.140)}$$

(on putting equation (B.36) for dz)

$$= -2\rho_f V_{fiss}\Omega_{ss}C_k'\int_{\lambda_{ss}}^{L}\frac{dC_{kss}}{\Omega_{ss}C_{kss}} - 2\rho_f^2 V_{fiss}\Omega_{ss}V_{gj}\int_{\lambda_{ss}}^{L}\frac{\rho_m'}{\rho_{mss}^2}\frac{dC_{kss}}{C_{kss}\Omega_{ss}} \qquad \text{(B.141)}$$

$$= -2\rho_f V_{fiss}C_k'\int_{\lambda_{ss}}^{L}\frac{dC_{kss}}{C_{kss}} - 2\rho_f^2 V_{fiss}V_{gj}\int_{\lambda_{ss}}^{L}\frac{\rho_m'}{\rho_f^2}\frac{C_{kss}^2}{C_{kss,\lambda ss}^2}\frac{dC_{kss}}{C_{kss}}$$

$$= -2\rho_f V_{fiss}C_k' \ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\frac{-2V_{fiss}V_{gj}}{C_{kss,\lambda ss}^2}\int_{\lambda_{ss}}^{L}C_{kss}\rho_m' dC_{kss} \qquad \text{(B.142)}$$

$$\int_{\lambda_{ss}}^{L} C_{kss}\rho'_m dC_{kss}$$

$$= \left[\frac{\Omega_{ss}}{s-\Omega_{ss}}\rho_f C_{kss,\lambda ss} C'_k \ln\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right] + \left[\left(\Omega_{ss}\lambda' - \frac{\Omega_{ss}}{s-\Omega_{ss}} C'_k\right)\rho_f \left(C_{kss,\lambda ss}\right)^{\frac{s}{\Omega_{ss}}}\right.$$

$$\left.\left(\frac{\Omega_{ss}}{\Omega_{ss}-s}\right)\left\{\left(C_{kss,L}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}} - \left(C_{kss,\lambda ss}\right)^{\frac{\Omega_{ss}-s}{\Omega_{ss}}}\right\}\right] = \mathrm{X} \qquad (B.137)$$

so $$\int_{\lambda_{ss}}^{L} 2V_{mss}V'_m \frac{\partial \rho_{mss}}{\partial z} dz = -2\rho_f V_{fiss} C'_k \ln\frac{Ck_{ss,L}}{Ck_{ss,\lambda ss}} - \frac{2V_{fiss}V_{gj}}{Ck^2_{ss,\lambda ss}}\mathrm{X} \qquad (B.143)$$

(6) Loss component

$$\Delta p_l = K_l \rho_m V_m^2 \qquad (B.144)$$

$$\Delta p_{lss} + \Delta p'_l = K_l\left(\rho_{mss} + \rho'_m\right)\left(V_{mss} + V'_m\right)^2 = K_l\left(\rho_{mss} + \rho'_m\right)\left(V^2_{mss} + 2V_{mss}V'_m\right)$$

$$= K_l \rho_{mss} V^2_{mss} + 2K_l \rho_{mss} V_{mss} V'_m + K_l \rho'_m V^2_{mss}$$

From equation (B.144) for steady state condition

$$\Delta p_{lss} = K_l \rho_{mss} V^2_{mss} \qquad (B.145)$$

so $$\Delta p'_l = 2K_l \rho_{mss} V_{mss} V'_m + K_l \rho'_m V^2_{mss} \qquad (B.146)$$

on putting the value of $V^2{}_{mss}$, we get

$$= 2K_l \rho_{mss} V_{mss} V'_m + K_l \rho'_m \left(\frac{\rho^2{}_{fiss} V^2{}_{fiss}}{\rho^2{}_{mss}}\right)$$

Now, on putting the value of ρ_{mss} from equation (B.40), we get

$$\Delta p'_l = 2K_l \rho_{fiss} V_{fiss} V'_{m(L,s)} + K_l \rho'_{m(L,s)} V^2_{fiss} \left(\frac{C_{kss,L}}{C_{kss,\lambda ss}}\right)^2 \qquad (B.147)$$

B.9 (b) Perturbed pressure drop for riser (unheated 2-ϕ region)

(1) Gravitational component

$$\Delta p_g = \int_L^{L_1} \rho_m g dz \tag{B.148}$$

on perturbing both sides of equation (B.148) we get

$$\Delta p_{gss} + \Delta p'_g = \int_L^{L_1} (\rho_{mss} + \rho'_m) g dz$$

or
$$\Delta p_{gss} + \Delta p'_g = \int_L^{L_1} \rho_{mss}\, g dz + \int_L^{L_1} \rho'_m\, g dz \tag{B.149}$$

for steady state condition equation (B.148) can be written as

$$\Delta p_{gss} = \int_L^{L_1} \rho_{mss}\, g dz \tag{B.150}$$

from equation (B.149) and equation (B.150), we get

$$\Delta p'_g = \int_L^{L_1} \rho'_m\, g dz$$

on putting the value of ρ'_m from equation (B.57), we get

$$= g \int_L^{L_1} \rho'_{m,L}\, e^{\frac{s(z-L)}{C_{kss,L}}}\, dz$$

$$\Delta p'_g = \rho'_{m,L} g \left(-\frac{C_{kss,L}}{s} \right) \left\{ e^{\frac{s(L_1-L)}{C_{kss,L}}} - 1.0 \right\} \tag{B.151}$$

(2) Frictional component

$$\Delta p_f = \int_L^{L_1} \frac{f_m}{2D} \rho_m V_m^2\, dz \tag{B.152}$$

perturbing both sides, we get

$$\Delta p_{fss} + \Delta p'_f = \int^{L1} \frac{f_m}{2D}(\rho_{mss} + \rho'_m)(V_{mss} + V'_m)^2 \, dz$$

$$= \int^{L_1} \frac{f_m}{2D}(\rho_{mss} + \rho'_m)(V_{mss}^2 + 2V_{mss}V'_m)dz$$

$$\Delta p_{fss} + \Delta p'_f = \int^{L_1} \frac{f_m}{2D} V_{mss}^2 dz + \int^{L_1} 2\rho_{mss} \frac{f_m}{2D} V_{mss} V'_m dz$$

$$+ \int^{L_1} \rho'_m \frac{f_m}{2D} V_{mss}^2 dz \qquad \text{(B.153)}$$

for steady state condition equation (B.152) can be written as

$$\Delta p_{fss} = \int^{L_1} \left(\frac{f_m}{2D}\right) \rho_{mss} V_{mss}^2 dz \qquad \text{(B.154)}$$

from equation (B.153) and (B.154), we get

$$\Delta p'_f = \int^{L_1} \frac{f_m}{D}(\rho_{mss} V_{mss}) V'_m dz + \int^{L_1} \frac{f_m}{2D} \rho'_m V_{mss}^2 dz$$

$$= \frac{f_m}{D}(\rho_{mss} V_{mss}) \int^{L_1} V'_m dz + \frac{f_m}{2D} V_{mss}^2 \int^{L_1} \rho'_m dz$$

or on putting value of V'_m and ρ'_m from equation (B.61) and (B.57) respectively, we get

$$\Delta p'_f = \frac{f_m}{D}(\rho_{mss} V_{mss}) \int^{L_1} \left[C'_k + \frac{\rho_f V_{gj} \rho'_{m,L} \, e^{-\frac{s(z-L)}{C_{kss,L}}}}{\rho_{mss}^2} \right] dz$$

$$+ \frac{f_m}{D} V_{mss}^2 \int^{L_1} \rho'_{m,L} \, e^{-\frac{s(z-L)}{C_{kss,L}}} dz$$

or $$\Delta p'_f = \frac{f_m}{D}(\rho_{mss} V_{mss}) \left[C'_k (L_1 - L) + \frac{\rho_f V_{gj} \rho'_{m,L} \left(-\frac{C_{kss}}{s}\right)}{\rho_{mss}^2} \right.$$

$$\left\{e^{\frac{s(L_1-L)}{C_{kss,L}}}-1.0\right\}\Bigg]\Bigg]+\left[\frac{f_m}{2D}V_{mss}^2\rho'_{m,L}\left(-\frac{C_{kss}}{s}\right)\quad\left\{e^{\frac{s(L_1-L)}{C_{kss}}}-1.0\right\}\right]\tag{B.155}$$

(1) Slip component

$$\Delta p_s=\int_L^{L_1}\frac{\partial}{\partial z}\left(\frac{\rho_f-\rho_m}{\rho_m-\rho_g}\frac{\rho_f\rho_g}{\rho_m}V_{gj}^2\right)dz$$

or
$$\Delta p_s=\left[\frac{\rho_f-\rho_m}{\rho_m-\rho_g}\frac{\rho_f\rho_g}{\rho_m}V_{gj}^2\right]_L^{L_1}\tag{B.156}$$

On perturbing both sides, we get

$$\Delta p_s+\Delta p'_s=\left[\frac{\rho_f-\rho_{mss}-\rho'_m}{\left(\rho_{mss}+\rho'_m\right)^2}\rho_f\rho_g V_{gj}^2\right]_L^{L_1}\tag{B.157}$$

since $\rho_m >> \rho_g$

$$=\left[\frac{\rho_f-\rho_{mss}-\rho'_m}{\left(\rho_{mss}^2+2\rho_{mss}\rho'_m\right)}\rho_f\rho_g V_{gj}^2\right]_L^{L_1}$$

$$=\left[\frac{\rho_f-\rho_{mss}-\rho'_m}{\rho_{mss}^2}\left(1-\frac{2\rho'_m}{\rho_{mss}}\right)\rho_f\rho_g V_{gj}^2\right]_L^{L_1}$$

$$=\left[\frac{\rho_f-\rho_{mss}}{\rho_{mss}^2}\left(1-\frac{2\rho'_m}{\rho_{mss}}\right)\rho_f\rho_g V_{gj}^2\right]_L^{L_1}$$

$$-\left[\frac{\rho'_m}{\rho_{mss}^2}\left(1-\frac{2\rho'_m}{\rho_{mss}}\right)\rho_f\rho_g V_{gj}^2\right]_L^{L_1}\tag{B.158}$$

now for steady state condition of equation (B.156)

$$\Delta p_{s,ss}=\left[\frac{\rho_f-\rho_{mss}}{\rho_{mss}-\rho_g}\frac{\rho_f\rho_g}{\rho_{mss}}V_{gj}^2\right]_L^{L_1}$$

On neglecting 2nd order term

$$\Delta p_i' = \int_L^{L_1} \rho_{mss} \frac{\partial V_m'}{\partial t} dz \qquad \text{(B.163)}$$

$$= \rho_{mss} s \int_L^{L_1} V_m' dz$$

on putting the value of V_m' from equation (B.61)

$$\Delta p_i' = s\rho_{mss} \int_L^{L_1} \left[C'_{k(L,s)} + \frac{\rho_f V_{gj} \rho'_{m,L} e^{\frac{s(z-L)}{C_{kss\,L}}}}{\rho^2_{mss,L}} \right] dz$$

$$\Delta p_i' = s\rho_{mss} \left[C'_{k(L,s)} (L_1 - L) + \frac{\rho_f V_{gj} \rho'_{m,L}}{\rho^2_{mss,L}} \left(-\frac{C_{kss,L}}{s} \right) \left\{ e^{\frac{s(L_1-L)}{C_{kss,L}}} - 1.0 \right\} \right]$$

(B.164)

5. Acceleration component

$$\Delta p_a = \int_L^{L_1} \rho_m V_m \frac{\partial V_m}{\partial Z} dz \qquad \text{(B.165)}$$

now from conservation of mass equation

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial}{\partial z}(\rho_m V_m) = 0$$

or $$\rho_m \frac{\partial V_m}{\partial z} = -\frac{\partial \rho_m}{\partial t} - V_m \frac{\partial \rho_m}{\partial z} \qquad \text{(B.166)}$$

on putting equation (B.166) into equation (B.165)

$$\Delta p_a = \int_L^{L_1} V_m \left(-\frac{\partial \rho_m}{\partial t} - V_m \frac{\partial \rho_m}{\partial z} \right) dz$$

or $$\Delta p_a = -\int_L^{L_1} V_m \frac{\partial \rho_m}{\partial t} dz - \int_L^{L_1} V_m^2 \frac{\partial \rho_m}{\partial z} dz \qquad \text{(B.167)}$$

on perturbing equation (B.167), we get

$$\Delta p_{ass} + \Delta p'_a = -\int_L^{L_1} (V_{mss} + V'_m) \frac{\partial}{\partial t} (\rho_{mss} + \rho'_m) dz$$

$$- \int_L^{L_1} (V_{mss} + V'_m)^2 \frac{\partial}{\partial z} (\rho_{mss} + \rho'_m) dz \tag{B.168}$$

$$\Delta p_{ass} + \Delta p'_a = -\int_L^{L_1} V_{mss}\, s\rho'_m dz - \int_L^{L_1} V_{mss}^2 \frac{\partial \rho_{mss}}{\partial z} dz$$

$$- \int_L^{L_1} V_{mss}^2 \frac{\partial \rho'_m}{\partial z} dz - \int_L^{L_1} 2V_{mss} V'_m \frac{\partial \rho_{mss}}{\partial z} dz \tag{B.169}$$

now for steady state condition of equation (B.52)

$$C_{kss} \frac{\partial \rho_{mss}}{\partial z} = 0$$

or $$\frac{\partial \rho_{mss}}{\partial z} = 0 \tag{B.170}$$

Also for steady state condition of equation (B.167), we get

$$\Delta p_{ass} = -\int_L^{L_1} V_{mss}^2 \frac{\partial \rho_{mss}}{\partial z} dz \tag{B.171}$$

on putting equation (B.170) and equation (B.171) into equation (B.169), we get

$$\Delta p'_a = -\int_L^{L_1} V_{mss}\, s\rho'_m dz - \int_L^{L_1} V_{mss}^2 \frac{\partial \rho'_m}{\partial z} dz$$

$$= -\int_L^{L_1} V_{mss}\, s\, \rho'_{m,L} e^{\frac{s(z-L)}{C_{kss,L}}} - \int_L^{L_1} V_{mss}^2 \frac{\partial \rho'_m}{\partial z} dz$$

$$= -V_{mss} s\rho'_{m,L} \left(-\frac{C_{kss,L}}{s} \right) \left\{ e^{\frac{s(L_1-L)}{C_{kss,L}}} - 1.0 \right\}$$

$$-V_{mss}^2\int_L^{L_1}\left(-\frac{s\rho_m'}{C_{kss}}\right)dz$$

$$=-V_{mss}s\rho_m'\left(-\frac{C_{kss,L}}{s}\right)\left\{e^{-\frac{s(L_1-L)}{C_{kss,L}}}-1.0\right\}$$

$$+\frac{V_{mss}^2 s}{C_{kss}}\int_L^{L_1}\rho_m' dz$$

$$=V_{mss}\rho_{m,L}'C_{kss,L}\left\{e^{-\frac{s(L_1-L)}{C_{kss,L}}}-1.0\right\}$$

$$+\frac{V_{mss}^2 s}{Ck_{ss,L}}\int_L^{L_1}\rho_{m,L}'e^{\frac{s(z-L)}{C_{kss,L}}}dz$$

$$=V_{mss}\rho_{m,L}'C_{kss,L}\left\{e^{-\frac{s(L_1-L)}{C_{kss,L}}}-1.0\right\}+\left[\frac{V_{mss}^2\rho_{m,L}'}{C_{kss,L}}\left(-\frac{C_{kss,L}}{s}\right)\left\{e^{-\frac{s(L_1-L)}{C_{kss\,L}}}-1.0\right\}\right]$$

or $$\Delta p_a'=\left\{e^{-\frac{s(L_1-L)}{C_{kss,L}}}-1\right\}\left\{V_{mss}\rho_{m,L}'C_{kss,L}-V_{mss}^2\rho_{m,L}'\right\} \quad \text{(B.172)}$$

6. Loss term

$$\Delta p_l=K_l\rho_m V_m^2 \quad \text{(B.173)}$$

now in the same way as we obtained for core region, we can get

$$\Delta p_l'=2K_l\rho_{mss}V_{mss}V_{m,L_1}'+K_l\rho_{m,L_1}'V_{mss}^2 \quad \text{(B.174)}$$